Kind of proposal (Talk)
What "is" the Yoneda Lemma?

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The "Yoneda Embedding" is easy to remember, but to prove it we need to prove the "Yoneda Lemma" first; the "Yoneda Lemma" takes a lot of mental space.

Let me refer to that "Yoneda Lemma" as the "Yoneda Lemma (in the strict sense)"; the "Yoneda Lemma (in the wide sense)" will be the "Yoneda Lemma (in the strict sense)" plus the "Yoneda Embedding". In the next paragraphs I will use the term"Yoneda Lemma" to mean the "Yoneda Lemma (in the wide sense)".

What "is" the Yoneda Lemma? Let's call that question Q1, and let's split it into several subquestions in the usual way:

Q2 What is the best notation, and what are the best diagrams, for remembering the Yoneda Lemma after we understand it well?

Q3 What are the best motivating examples for the Yoneda Lemma? As an aside, what does "best motivating examples" mean? Can we formalize that idea? There is a partial answer in [1]; are there other ones?

Q4 Imagine that you want to present the Yoneda Lemma for non-categorists, and imagine that you can prepare a video for your presentation, and complement it with a blog post. What are the figures and animations that you would choose? Which slogans would you choose? Why?

Note that I am sort of splitting "what is the Yoneda Lemma?" into several subquestions, and some of them are naturally associated to verbs: "remember", "reconstruct", "generalize", "specialize", "present", "teach"; some others are nouns: "analogies", "animations", "slogans".

I only started studying Mathematical Education ("ME") very recently, and I was very surprised to see that this division into subquestions is studied a lot in ME. One extreme example is the book [2], that discusses "deep understanding" in the context of teaching Arithmetic to children, but practically all the ideas in that book are trivial to adapt to Category Theory. A less extreme example is [3], that discusses "levels of thinking", mainly in the context of school geometry.

In my presentation I will show some texts and ideias from ME that I believe that would be especially useful to categorists. The "recommended readings" will be basically the references at the end of [4].

References

[1] E. Ochs. *On the Missing Diagrams in Category Theory (First-Person Version).* `https://anggtwu.net/math-b.html#2022-md`. 2022. arXiv: 2401.12345.

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[2] L. Ma. *Knowing and Teaching Elementary Mathematics (Anniversary Edition).* Routledge, 2010.

[3] P. M. van Hiele. *Structure and Insight - A Theory of Mathematics Education.* Academic Press, 1986.

[4] E. Ochs. *Bad Foundations and Manipulable Objects.* `https://anggtwu.net/math-b.html#2026-bad-foundations`. 2026. arXiv: 2602.16026.