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# A Revolution in One Classroom: The Case of Mrs. Oublier

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*This essay probes the relationship between instructional policy and teaching practice. In the mid 1980s, California State officials launched an ambitious effort to revise mathematics teaching and learning. The aim was to replace mechanical memorization with mathematical understanding. This essay considers one teacher's response to the new policy. She sees herself as a success for the policy: she believes that she has revolutionized her mathematics teaching. But observation of her classroom reveals that the innovations in her teaching have been filtered through a very traditional approach to instruction. The result is a remarkable melange of novel and traditional material. Policy has affected practice in this case, but practice has had an even greater effect on policy.*

As Mrs. Oublier sees it, her classroom is a new world. She reported that when she began work 4 years ago, her mathematics teaching was thoroughly traditional. She followed the text. Her second graders spent most of their time on worksheets. Learning math meant memorizing facts and procedures. Then Mrs. O found a new way to teach math. She took a workshop in which she learned to focus lessons on students' understanding of mathematical ideas. She found ways to relate mathematical concepts to students' knowledge and experience. And she explored methods to engage students in actively understanding mathematics. In her third year of such work, Mrs. O was delighted with her students' performance, and with her own accomplishments.

Mrs. O's story is engaging, and so is she. She is considerate of her students, eager for them to learn, energetic, and attractive. These qualities would stand out anywhere, but they seem particularly vivid in her school. It is a drab collection of one-story, concrete buildings that sprawl over several acres. Though clean and well managed, her school lacks any of the familiar signs of innovative education. It has no legacy of experimentation or progressive pedagogy, or even of heavy spending on education. Only a minority of children come from well-to-do families. Most families have middling or modest incomes, and many are eligible for Chapter 1 assistance. A sizable minority are on welfare. The school district is situated in a dusty corner of southern California, where city migrants rapidly are turning a rural town into a suburb. New condominiums are sprouting all over the community, but one still sees pickup trucks with rifle racks mounted in their rear windows. Like several of her colleagues, Mrs. O works in a covey of portable, prefab classrooms, trucked into the back of the schoolyard to absorb growing enrollments.

Mrs. O's story seems even more unlikely when considered against the history of American educational reform. Great plans for educational change are a familiar feature of that history, but so are reports of failed reforms. That is said to have been the fate of an earlier "new math" in the 1950s and 1960s. A similar tale was told of efforts to improve science teaching at the time (Welsh, 1979). Indeed, failed efforts to improve teaching and learning are an old story. John Dewey and others announced a revolution in pedagogy just as our century opened, but apparently it fizzled: Classrooms changed only a little, researchers say (Cuban, 1984). The story goes on. Since the Sputnik era, many studies of instructional innovation have embroidered these old themes of great ambitions and modest results (Gross, Giaquinta, & Bernstein, 1971; Rowan & Guthrie, 1989; Cohen, 1989).

Some analysts explain these dismal tales with reference to teachers' resistance to change: They argue that entrenched classroom habits defeat reform (Gross, Giaquinta, & Bernstein, 1971). Others report that many innovations fail because they are poorly adapted to classrooms: Even teachers who avidly desire change can do little with most schemes to improve instruction, because they don't work well in classrooms (Cuban, 1984, 1986). Mrs. O's revolution looks particularly appealing against this background. She eagerly embraced change, rather than resisting it. She found new ideas and materials that worked in her classroom, rather than resisting innovation. Mrs. O sees her class as a success for the new mathematics framework. Though her revolution began while the framework was still being written, it was inspired by many of the same ideas. She reports that her math teaching has wound up where the framework intends it to be.

Yet as I watched and listened in Mrs. O's classroom, things seemed more complicated. Her teaching does reflect the new framework in many ways. For instance, she had adopted innovative instructional materials and activities, all designed to help students make sense of mathematics. But Mrs. O seemed to treat new mathematical topics as though they were a part of traditional school mathematics. She used the new materials, but used them as though mathematics contained only right and wrong answers. She has revised the curriculum to help students understand math, but she conducts the class in ways that discourage exploration of students' understanding.

From the perspective of the new mathematics framework, then, Mrs. O's lessons seem quite mixed. They contain some important elements that the framework embraced, but they contain others that it branded as inadequate. In fact, her classes present an extraordinary melange of traditional and novel approaches to math instruction.

### Something Old and Something New

That melange is part of the fascination of Mrs. O's story. Some observers would agree that she has made a revolution, but others would see only traditional instruction. It is easy to imagine long arguments about which is the real Mrs. O, but they would be the wrong arguments. Mrs. O. is both of these teachers. Her classroom deserves attention partly because such mixtures are quite common in instructional innovations—though they have been little noticed. As teachers and students try to find their way from familiar practices to new ones, they cobble new ideas onto familiar practices. The variety of these blends and teachers' ingenuity in fashioning them are remarkable, but they raise unsettling questions. Can we say that an innovation has made such progress when it is tangled in combination with many traditional practices? Changes that seem large to teachers who are in the midst of struggles to accommodate new ideas often seem modest or invisible to observers who scan practice for evidence that new policies have been implemented. How does one judge innovative progress? Should we consider changes in teachers' work from the perspective of new policies like the framework? Or should they be considered from the teachers' vantage point?

#### *New Mathematics, Old Mathematics*

From one angle, the curriculum and instructional materials in this class were just what the new framework ordered. For instance, Mrs. O regularly asked her second graders to work on "number sentences." In one class that I observed, students had done the problem: $10 + 4 = 14$. Mrs. O then asked them to generate additional number sentences about 14. They volunteered various ways to write addition problems about fourteen—that is, $10 + 1 + 1 + 1 + 1 = 14$, $5 + 5 + 4 = 14$, and so forth. Some students proposed several ways to write subtraction problems—that is, 14 − 4 = 10, 14 − 10 = 4, and so forth. Most of the students' proposals were correct. Such work could make mathematics relationships more accessible by coming at them with ordinary language rather than working only with bare numbers on a page. It also could unpack mathematics relationships by offering different ways to get the same result. It could illuminate the relations between addition and subtraction, helping children to understand their reversibility. And it could get students to do "mental math," that is, to solve problems in their heads and thereby learn to see math as something to puzzle about and figure out, rather than just a bunch of facts and procedures to be memorized.

These are all things that the new framework invited. The authors exhort teachers to help students cultivate " . . . an attitude of curiosity and the willingness to probe and explore . . . " (California State Department of Education [CSDE], 1985, p. 1). The document also calls for classroom work that helps students " . . . to understand why computational algorithms are constructed in particular forms . . . " (p. 4).

Yet the framework's mathematical exhortations were general; it offered few specifics about how teachers might respond, and left room for many different responses. Mrs. O used the new materials, but conducted the entire exercise in a thoroughly traditional fashion. The class worked as though the lesson were a drill, reciting in response to the teacher's queries. Students' sentences were accepted if correct, and written down on the board. They were turned down if incorrect, and not written on the board. Right answers were not explained, and wrong answers were treated as unreal. The framework makes no such distinction. To the contrary, it argues that understanding how to arrive at answers is an essential part of helping students to figure out how mathematics works—perhaps more important than whether the answers are right or wrong. The framework criticizes the usual memorized, algorithmic approach to mathematics, and the usual search for the right answer. It calls for class discussion of problems and problem solving as an important part of figuring out mathematical relationships (CSDE, 1985, pp. 13–14). But no one in Ms. O's class was asked to explain their proposed number sentences, correct or incorrect. No student was invited to demonstrate how he or she knew whether a sentence was correct or not. The teacher used a new mathematics curriculum, but used it in a way that conveyed a sense of mathematics as a fixed body of right answers, rather than as a field of inquiry in which people figure out quantitative relations. It is easy to see the framework's ideas in Mrs. O's classroom, but it also is easy to see many points of opposition between the new policy and Mrs. O's approach (CSDE, 1987, p. 9).

Make no mistake: Mrs. O was teaching math for understanding. The work with number sentences certainly was calculated to help students see how addition worked, and to see that addition and subtraction were reversible. That mathematical idea is well worth understanding, and the students seemed to understand it at some level. They were, after all, producing the appropriate sorts of sentences. Yet it was difficult to understand how or how well they understood it, for the didactic form of the lesson inhibited explanation or exploration of students' ideas. Additionally, mathematical knowledge was treated in a traditional way: Correct answers were accepted, and wrong ones simply rejected. No answers were unpacked. There was teaching for mathematical understanding here, but it was blended with other elements of instruction that seemed likely to inhibit understanding.

The mixture of new mathematical ideas and materials with old mathematical knowledge and pedagogy permeated Mrs. O's teaching. It also showed up extensively in her work with concrete materials and other physical activities. These materials and activities are a crucial feature of her revolution, for they are intended to represent mathematical concepts in a form that is vivid and accessible to young children. For instance, she opens the math lesson every day with a calendar activity, in which she and the students gather on a rug at one side of the room to count up the days of the school year. She uses this activity for various purposes. During my first visit she was familiarizing students with place value, regrouping, and odd and even numbers. As it happened, my visit began on the fifty-ninth day of the school year, and so the class counted to fifty-nine. They used single claps for most numbers but double claps for ten, twenty, and so on. Thus, one physical activity represented the "tens", and distinguished them from another physical activity that was used to represent the "ones." On the next day, the class used claps for even numbers and finger snaps for odd numbers, in counting off the days. The idea here is that fundamental distinctions among types of number can be represented in ways that make immediate and fundamental sense to young children. Representations of this sort, it is thought, will deeply familiarize them with important mathematical ideas, but will do so in a fashion easily accessible to those unfamiliar with abstractions.

Mrs. O also used drinking straws in a related activity, to represent place value and regrouping, Every day a "student helper" is invited to help lead the calendar activity by adding another straw to the total that represent the elapsed days in the school year. The straws accumulate until there are ten, and then are bundled with a rubber band. One notion behind this activity is that students will gain some concrete basis for understanding how numbers are grouped in a base ten system. Another is that they can begin to apprehend, first physically and then intellectually, how number groups can be composed and decomposed.

Mrs. O's class abounds with such activities and materials, and they are very different from the bare numbers on worksheets what would be found in a traditional math class. She was still excited, after several years' experience, about the difference that they made for her students' understanding of arithmetic. Mrs. O adopts a somewhat cool demeanor in class. However, her conviction about the approach was plain, and her enthusiasm for it bubbled up in our conversations. After 3 years, she had only disdain for her old way of teaching math.

Her approach seems nicely aligned with the new framework. For instance, that document argues that "many activities should involve concrete experiences so that students develop a sense of what numbers mean and how they are related before they are asked to add, subtract, multiply, or divide them" (CSDE, 1985, p. 8). And it adds, a few pages further on, that "Concrete materials provide a way for students to connect their own understandings about real objects and their own experiences to mathematical concepts. They gain direct experience with the underlying principles of each concept" (p. 15).

Mrs. O certainly shared the framework's view in this matter, but it is one thing to embrace a doctrine of instruction, and quite another to weave it into one's practice. For even a rather monotonous practice of teaching comprises many different threads. Hence any new instructional thread must somehow be related to many others already there. Like reweaving fabric, this social and intellectual reweaving can be done in different ways. The new thread can simply be dropped onto the fabric, and everything else left as is. Or new threads may be somehow woven into the fabric. If so, some alteration in the relations among threads will be required. Some of the existing threads might have to be adjusted in some way, or even pulled out and replaced. If one views Mrs. O's work from the perspective of the framework, new threads were introduced, but old threads were not pulled out. The old and new lay side by side, and so the fabric of instruction was different. However, there seemed to be little mutual adjustment among new and old threads. Mrs. O used the novel concrete materials and physical activities, but used them in an unchanged pedagogical surrounding. Consequently the new material seemed to take on different meaning from its circumstances. Materials and activities intended to teach mathematics for understanding were infused with traditional messages about what mathematics was, and what it meant to understand it.

These mixed qualities were vividly apparent in a lesson that focused on addition and subtraction with regrouping. The lesson occurred early in an 8- or 10-week cycle concerning these topics. Like many of her lessons, it combined a game-like activity with the use of concrete materials. The aim was to capture children's interest in math, and to help them understand it. Mrs. O introduced this lesson by announcing: "Boys and girls, today we are going to play a counting game. Inside this paper [holding up a wadded up sheet of paper] is the secret message . . . " (observation notes, December 5, 1988). Mrs. O unwadded the paper and held it up: "6" was inscribed. The number was important, because it would establish the number base for the lesson: Six. In previous lessons they had done the same thing with four and five. So part of the story here was exploring how things work in different number bases, and one reason for that, presumably, was to get some perspective on the base-ten system that we conventionally use. Mrs. O told the children that, as in the previous games, they would use a nonsense word in place of the secret number. I was not sure why she did this, at the time. As it turned out, the approach was recommended, but not explained, by the innovative curriculum guide she was using. After a few minutes taken to select the nonsense word, the class settled on "Cat's eye." (observation notes, December 5, 1988. These notes are the source of the remainder of this episode).

With this groundwork laid, Mrs. O had "place value boards" given to each student. She held her board up [eight by eleven, roughly, one half blue and the other white], and said: "We call this a place value board. What do you notice about it?"

Cathy Jones, who turned out to be a steady infielder on Mrs. O's team, said: "There's a smiling face at the top." Mrs. O agreed, noting that the smiling face needed to be at the top at all times [that would keep the blue half of the board on everyone's left]. Several kids were holding theirs up for inspection from various angles, and she admonished them to leave the boards flat on their tables at all times.

"What else do we notice?" she inquired. Sam said that one half is blue and the other white. Mrs. O agreed, and went on to say that " . . . the blue side will be the "cat's eye" side. During this game we will add one to the white side, and when we get a cat's eye, we will move it over to the blue side." With that, each student was given a small plastic tub, which contained a handful of dried beans and half a dozen small paper cups, perhaps a third the height of those dispensed in dentist's offices. This was the sum total of prelesson framing–no other discussion or description preceded the work.

There was a small flurry of activity as students took their tubs and checked out the contents. Beans present nearly endless mischievous possibilities, and several of the kids seemed on the verge of exploring their properties as guided missiles. Mrs. O nipped off these investigations, saying: "Put your tubs at the top of your desks, and put both hands in the air." The students all complied, as though in a small stagecoach robbery. "Please keep them up while I talk." She opened a spiral bound book, not the school district's adopted text but *Math Their Way* (Baratta-Lorton, 1976). This was the innovative curriculum guide that had helped to spark her revolution. She looked at it from time to time, as the lesson progressed, but seemed to have quite a good grip on the activity.

Mrs. O got things off to a brisk start: "Boys and girls [who still were in the holdup], when I clap my hands, add a bean to the white side [from the plastic tub]."

She clapped once, vigorously, adding that they could put their hands down. "Now we are going to read what we have: What do we have?" [she led a choral chant of the answer] "Zero cat's eye and one." She asked students to repeat that, and everyone did. She clapped again, and students obediently added a second bean to the white portion of the card. "What do we have now," she inquired. Again she led a choral chant: "Zero cat's eye and two." So another part of the story in this lesson was place value: "Zero cat's eye" denotes what would be the "tens" place in base-ten numbering, and "two" is the "one's" place. Counting individual beans, and beans grouped in "cat's eye," would give the kids a first-hand, physical sense of how place value worked in this and other number bases.

In these opening chants, as in all subsequent ones, Mrs. O performed more vigorously than rythmically. Rather than establishing a beat and then maintaining it with her team, she led each chant and the class followed at a split-second interval. Any kid who didn't grasp the idea needed only to wait for her cue, or for his table-mates. There were no solos: Students were never invited or allowed to count on their own. Thus, although the leitmotif in their second chant was "zero cat's eye and two," there was an audible minor theme of "zero cat's eye and one." That several repeated the first chant suggested that they did not get either the routine or its point.

Mrs. O moved right on nonetheless, saying that it " . . . is very important that you read the numbers with your hands." This was a matter to which she returned many times during the lesson; she kept reminding the children to put their little hands first on the beans on the white square, and then on the little cups on the blue square, as they incanted the mathematical chants. It was essential that they manipulated the concrete materials. Whenever she spotted children who were not palpitating beans and cups, she walked over and moved their arms and hands for them.

Mrs. O led the bean adding and chants up to five. Then, when the first five beans were down on everyone's card, she asked: "Now think ahead; when I clap my hands this time, what will you have on the white side?"

Reliable Cathy Jones scooped it up and threw smoothly to first: "Cat's eye."

Mrs. O led off again: "When you get a cat's eye, put all the beans in a paper cup, and move them over." She clapped her hands for the cat's eye, and then led the following chant: "Put the beans in the cup and move them over."

"Now let's read what we have." The chant rolled on, "one cat's eye and zero." A puzzling undercurrent of "one cat's eye and one" went unattended. She then led the class through a series of claps and chants, leading up to two cat's eyes. And the claps and chants went on, with a methodical monotony, up to five cat's eyes and five. The whole series took about 15 minutes, and throughout the exercise she repeatedly reminded students to "read" the materials with their hands, to feel the beans and move their arms. By the time they got to five cat's eyes and five, her claps had grown more perfunctory, and many of the kids had gotten the fidgets, but Mrs. O gave no ground. She seemed to see this chanting and bean-handling as the high road to mathematical understanding, and tenaciously drove her team on.

"Now, how many do we have?" "Five cat's eyes and five beans," came the chant. "Now we will take away one bean [from the "ones" side of the board]. How many do we have?" Again the answering chant, again led by her, a fraction of a second early, "five cat's eyes and four."

This was a crucial point in the lesson. The class was moving from what might be regarded as a concrete representation of addition with regrouping, to a similar representation of subtraction with regrouping. Yet she did not comment on or explain this reversal of direction. It would have been an obvious moment for some such comment or discussion, at least if one saw the articulation of ideas as part of understanding mathematics. Mrs. O did not teach as though she took that view. Hers seemed to be an activity-based approach: It was as though she thought that all the important ideas were implicit, and better that way.

Thus the class counted down to five cat's eyes and zero. Mrs. O then asked, "What do we do now?" Jane responded: "Take a dish from the cat's eye side, and move it to the white side." No explanation was requested or offered to embroider this response. Mrs. O simply approved the answer, clapped her hands, and everyone followed Jane's lead. With this, Mrs. O led the class back through each step, with claps, chants, and reminders to read the beans with their hands, down to zero cat's eye and zero beans. The entire effort took 30 or 35 minutes. Everyone was flagging long before it was done, but not a chant was skipped or a movement missed.

Why did Mrs. O teach in this fashion? In an interview following the lesson I asked her what she thought the children learned from the exercise. She said that it helped them to understand what goes on in addition and subtraction with regrouping. Manipulating the materials really helps kids to understand math, she said. Mrs. O seemed quite convinced that these physical experiences *caused* learning, that mathematical knowledge arose from the activities.

Her immediate inspiration for all this seems to have been *Math Their Way,* a system of primary grade math teaching on which, Mrs. O says, she relies heavily. *Math Their Way* announces its purpose this way: " . . . to develop understanding and insight of the patterns of mathematics through the use of concrete materials" (Baratta-Lorton, 1976, p. xiv). Concrete materials and physical activities are the central features of this primary grade program, because they are believed to provide real experience with mathematics. In this connection the book sharply distinguishes between mathematical symbols and concepts. It criticizes teaching with symbols, arguing that symbols (i.e., numbers) " . . . are not *the concept,* they are only a representation of the concept, and as such are abstractions describing something which is not visible to the child. Real materials, on the other hand, can be manipulated to illustrate the concept concretely, and can be experienced visually by the child. . . . The emphasis throughout this book is making concepts, rather than numerical symbols, meaningful" (p. xiv).

*Math Their Way* fairly oozes with the belief that physical representations are much more real than symbols. This fascinating idea is a recent mathematical mutation of the belief, at least as old as Rousseau, Pestalozzi, and James Fenimore Cooper, that experience is a better teacher than mere books. For experience is vivid, vital, and immediate, whereas books are all abstract ideas and dead formulations. Mrs. O did not mention these sages, but she certainly had a grip on the idea. In this she resembles many primary school teachers, for the view that concrete materials and physical activities are the high road to abstract concepts has become common currency in nursery school and primary grade teaching. Many primary grade teachers have long used physical activities and concrete materials elsewhere in instruction.

In fact, one of the chief claims in Baratta-Lorton's book is that concrete materials are developmentally desirable for young children. Numbers are referred to many times as an "adult" way of approaching math. And this idea leads to another, still more important one: If math is taught properly, it will be easy. Activities with concrete materials, the book insists, are the natural way for kids to learn math: " . . . if this foundation is firmly laid, dealing with abstract number will be *effortless*" (p. 167, italics added).

Stated so baldly, that seems a phenomenal claim: Simply working with the proper activities and materials assures that math will be understood. Materials and activities are not only necessary for understanding mathematics, but also sufficient. But the idea is quite common. Pestalozzi might have cheered it. Many other pedagogical romantics, Rousseau and Dewey among them, embraced a version of this view. Piaget is commonly thought to have endorsed a similar idea. So when *Math Their Way* argues that the key to teaching math for understanding is to get children to use the right sorts of activities and materials, it is on one of the main tracks of modern educational thought and practice. The book's claim also helps to explain why it gives no attention to the nature of mathematical knowledge, and so little attention to the explanation of mathematical ideas. For the author seems convinced that such things are superfluous: Appropriate materials and activities alone will do the trick.

In fact, the book's appeal owes something to its combination of great promises and easy methods. It offers teachers a kind of pedagogical special, a two-for-the-price-of-one deal: Students will "understand" math without any need to open up questions about the nature of mathematical knowledge. The curriculum promises mathematical understanding, but it does not challenge or even discuss the common view of mathematics as a fixed body of material—in which knowledge consists of right answers—that so many teachers have inherited from their own schooling. The manual does occasionally note that teachers might discuss problems and their solutions with students, but this encouragement is quite modestly and intermittently scattered through a curriculum guide that chiefly focuses on the teaching potential of concrete materials and physical activities. The book presents concrete representations and math activities as a kind of explanation sufficient unto themselves. Discussion of mathematical ideas has a parenthetical role, at best.

All of this illuminates Mrs. O's indebtedness to *Math Their Way,* and her persistent praise for it. She used the guide to set up and conduct the lessons that I saw, and referred to it repeatedly in our conversations as the inspiration for her revolution. My subsequent comparisons of her classes with the manual suggested that she did draw deeply on it for ideas about materials, activities, and lesson format. More important, her views of how children come to understand mathematics were, by her own account, powerfully influenced by this book.

Baratta-Lorton's book thus enabled Mrs. O to whole-heartedly embrace teaching math for understanding, without considering or reconsidering her views of mathematical knowledge. She was very keen that children should understand math, and worked hard at helping them. However, she placed nearly the entire weight of this effort on concrete materials and activities. The ways that she used these materials—insisting, for instance, that all the children actually feel them, and perform the same prescribed physical operations with them—suggested that she endowed the materials with enormous, even magical instructional powers. The lack of any other ways of making sense of mathematics in her lesson was no oversight. She simply saw no need for anything else.

In what sense was Mrs. O teaching for understanding? The question opens up a great puzzle. Her classes exuded traditional conceptions of mathematical knowledge, and were organized as though explanation and discussion were irrelevant to mathematics. Yet she had changed her math teaching quite dramatically. She now used a new curriculum specially designed to promote students' understanding of mathematics. And her students' lessons were very different than they had been. They worked with materials that represented mathematical relationship in the concrete ways that the framework and many other authorities endorse. Mrs. O thought the change had been decisive: She now teaches for understanding. She reported that her students now understood arithmetic, whereas previously they had simply memorized it.

### *New Topics, Old Knowledge*

Mrs. O taught several topics endorsed by the new framework that would not have been covered in many traditional math classes. One such topic was estimation. Mrs. O told me that estimation is important because it helps students to make sense of numbers. They have to make educated guesses, and learn to figure out why some guesses are better than others. She reports that she deals with estimation recurrently in her second-grade classwork, returning to it many times in the course of the year rather than teaching a single unit. Her reason was that estimation could not be learned by doing it once or twice, and, in any event, is useful in many different problem-solving situations. Her reasoning on this matter seemed quite in accord with the framework. It calls for "guessing and checking the result" as an important element in mathematical problem solving [CSDE, 1985, p. 14). In fact, the framework devotes a full page to estimation, explaining what it is and why it is important (pp. 4–5).

The teaching that I observed did not realize these ambitions. In one lesson, for instance, the following problem was presented: Estimate how many large paper clips would be required to span one edge of the teacher's desk (observation notes, December 6, 1988). Two students were enlisted to actually hold up the clips so that students could see. They stood near the teacher's desk, near enough to visually gauge its width in relation to the clips, but all the other students remained at their tables, scattered around the room. None had any clips, and few could see the edge of the teacher's desk that was in question. It was a side edge, away from most of the class.

So only two members of the class had real contact with the two key data sources in the problem—visible, palpable clips, and a clear view of the desk edge. As a consequence, only these two members of the class had any solid basis for deciding if their estimates were mathematically reasonable. Even Mrs. O was seated too far away to see the edge well, and she had no clips either. The problem itself was sensible, and could have been an opportunity to make and discuss estimates of a real puzzle. Unfortunately, it was set up in a way that emptied it of opportunities for mathematical sense-making.

Mrs. O did not seem aware of this. After she had announced the problem, she went on to engage the whole class in solving it. The two students were told to hold the clips up for everyone to see. Seated at the back, with many of the kids, I could see that they were the large sort of clip, but even then they were barely visible. Mrs. O then pointed to the desk edge, at the other end of the room, easily 20 feet from half the class. Then she asked the students to estimate how many clips it would take to cover the edge, and to write down their answers. She took estimates from most of the class, wrote them on the board, and asked class members if the estimates were reasonable.

Not surprisingly, the answers lacked mathematical discrimination. Estimates that were close to three times the actual answer, or one third of it, were accepted by the class and the teacher as reasonable. Indeed, no answers were rejected as unreasonable, even though quite a few were far off the mark. Nor were some estimates distinguished as more or less reasonable than others. Mrs. O asked the class what reasonable meant, and one boy offered an appropriate answer, suggesting that the class had some previous contact with this idea.

There was nothing that I could see or imagine in the classroom that led inexorably to this treatment. Mrs. O seemed to have many clips. If eight or ten had been passed around, the kids would have had at least direct access to one element in the estimation problem—that is, the length of the clip. Additionally, Mrs. O could have directed the kids' attention to the edge of the desk that they could see, rather than the far edge that they could not see. I knew that the two edges of the rectangular desk were the same length, and perhaps some of these second graders did as well, but her way of presenting the problem left that as a needless, and mathematically irrelevant, barrier to their work. Alternatively, Mrs. O could have invited them to estimate the length of their own desk edges, which were all the same, standard-issue models. That, along with passed-around clips, would have given them much more direct contact with the elements of the problem. The students would have had more of the mathematical data required to make sound estimates, and much more of a basis for considering the reasonableness of those estimates.

Why did Mrs. O not set the problem up in one of these ways? I could see no organizational or pedagogical reason. In a conversation after the class, when I asked for her comments on that part of the lesson, she did not display even a shred of discomfort, let alone suggest that anything had been wrong. Mrs. O seemed to understand the broad purpose of teaching and learning estimation (interview, December 6, 1988). However, this bit of teaching suggests that she did not have a firm grip on the mathematics in this estimation example. She taught as though she lacked the mathematical and pedagogical infrastructure—the knowledge of mathematics, and of teaching and learning mathematics—that would have helped her to set the problem up so that the crucial mathematical data were available to students.

An additional bit of evidence on this point concerns the way Mrs. O presented estimation. She offered it as a topic in its own right, rather than as a part of solving problems that came up in the course of studying mathematics. After ending one part of the lesson, she turned to estimation as though it were an entirely separate matter. When the estimation example was finished, she turned the class to still another topic. Estimation had an inning all its own, rather than being woven into other innings' work. It was almost as though she thought that estimation bore no intimate relation to solving the ordinary run of mathematical problems. This misses the mathematical point: Estimation is useful and used in that ordinary run, not for its own sake. The framework touches on this matter, arguing that ". . . estimation activities should be presented not as separate lessons, but as a step to be used in all computational activities" (CSDE, 1985, p. 4).

When detached from regular problem solving, estimation may seem strange, and thus isolated may lose some of its force as a way of making sense in mathematics. I wondered what the students might have learned from this session. They all appeared to accept the lesson as reasonable. No students decried the lack of comprehensible data on the problem, which they might have done if they were used to such data, and if this lesson were an aberration. No one said that he or she had done it differently some other time, and that this didn't make sense. That could mean that the other lessons on estimation conveyed a similar impression. Or it may mean that students were simply dutiful, doing what they had been told because they had so often been told to do so. Or it may mean only that students took nothing from the lesson. Certainly school is full of mystifying or inexplicable experiences that children simply accept. Perhaps this struck them as another such mystification. It is possible, though, that they did learn something, and that it was related to Mrs. O's teaching. If so, perhaps they learned that estimation was worth doing, even if they didn't learn much about how to do it. Or perhaps they acquired an inappropriate idea of what estimation was, and what reasonable meant.

Was this teaching math for understanding? From one angle, it plainly was. Mrs. O did teach a novel and important topic, specifically intended to promote students' sense-making in arithmetic. It may well have done that. Yet the estimation problem was framed so that students had no way to bring mathematical evidence to bear on the problem, and little basis for making reasonable estimates. It therefore also is possible that students found this puzzling, confusing, or simply mysterious. These alternatives are not mutually exclusive. This bit of teaching for understanding could have promoted more understanding of mathematics, along with more misunderstanding.

### *New Organization, Old Discourse*

Mrs. O's class was organized to promote cooperative learning. The students' desks and tables were gathered in groups of four and five, so that they could easily work together. Each group had a leader, to help with various logistical chores. And the location and distribution of instructional materials often were managed by groups rather than individually. The new framework endorses this way of organizing classroom work. It puts the rationale this way: "To internalize concepts and apply them to new situations, students must interact with materials, express their thoughts, and discuss alternative approaches and explanations. Often, these activities can be accomplished well in groups of four students" (CSDE, 1985, p. 16).

The framework thus envisions cooperative learning groups as the vehicle for a new sort of instructional discourse, in which students would do much more of the teaching. In consequence, each of them would learn from their own efforts to articulate and explain ideas—much more than they could learn from a teacher's explanations to them. And they would teach each other as well, learning from their classmates' ideas and explanations, and from others' responses. The framework explains: "Students have more chances to speak in a small group than in a class discussion; and in that setting some students are more comfortable speculating, questioning, and explaining concepts in order to clarify their thinking" (CSDE, 1985, pp. 16–17).

Mrs. O's class was spatially and socially organized for such cooperative learning, but the instructional discourse that she established cut across the grain of this organization. The class was conducted in a highly structured and classically teacher-centered fashion. The chief instructional group was the whole class. The discourse that I observed consisted either of dyadic exchanges between the teacher and one student, or of whole-group activities, many of which involved choral responses to teacher questions. No student ever spoke to another about mathematical ideas as a part of the public discourse. Nor was such conversation ever encouraged by the teacher. Indeed, Mrs. O specifically discouraged students from speaking with each other, in her efforts to keep the class orderly and quiet.

The small groups were not ignored. They were used for instructional purposes, but they were used in a distinctive way. In one class that I observed, for instance, Mrs. O announced a "graphing activity" about midway through the math period. She wrote across the chalk board, at the front of the room *Letter to Santa?* Underneath she wrote two column headings: *Yes* and *No*. Then she told the children that she would call on them by groups, to answer the question.

If she had been following the framework's injunctions about small groups, Mrs. O might have asked each group to tally its answers to the question. She might then have asked each group to figure out whether it had more yes than no answers, or the reverse. She might then have asked each group to figure out how many more. And she might have had each group contribute its totals to the chart at the front of the room. This would not have been the most challenging group activity, but it would have meaningfully used the small groups as agents for working on this bit of mathematics.

Mrs. O proceeded differently. She used the groups to call on individual children. Moving from her right to left across the room, she asked individuals from each group, seriatim, to come to the front and put their entry under the Yes or No column, exhausting one group before going on to the next. The groups were used in a socially meaningful way, but there was no mathematical discourse within them.

Mrs. O used the small groups in this fashion several times during my visits. The children seemed quite familiar with the procedures, and worked easily in this organization. In addition, she used the groups to distribute and collect instructional materials, which was a regular and important feature of her teaching. Finally, she regularly used the groups to dismiss the class for lunch and recess: She would let the quietest and tidiest group go first, and so on through the class.

Small groups thus were a regular feature of instruction in Mrs. O's class. I asked her about cooperative grouping in one of our conversations: Did she always use the groups in the ways that I had observed? She thought she did. I asked if she ever used them for more cooperative activity, that is, discussions and that sort of thing. She said that she occasionally did so, but mostly she worked in the ways I had observed.

In what sense was this teaching or understanding? Here again, there was a remarkable combination of old and new math instruction. Mrs. O used a new form of classroom organization that was designed to promote collaborative work and broader discourse about academic work. She treated this organization with seriousness. She referred to her classwork as "cooperative learning," and used the organization for some regular features of classroom work. When I mentally compared her class with others I had observed, in which students sat in traditional rows, and in which there was only whole-group or individual work, her class seemed really different. Though Mrs. O runs a tight ship, her class was more relaxed than those others I remembered, and organized in a more humane way. My view on this is not simply idiosyncratic. If Cuban had used this class in the research for *How Teachers Taught* (1984), he probably would have judged it to be innovative as well. For that book relies on classrooms' social organization as an important indicator of innovation.

Mrs. O also judged her classroom to be innovative. She noted that it was now organized quite differently than during her first year of teaching, and she emphatically preferred the innovation. The kids were more comfortable, and the class much more flexible, she said. Yet she filled the new social reorganization of discourse with old discourse processes. The new organization opened up lots of new opportunities for small group work, but she organized the discourse in ways that effectively blocked realization of those opportunities.

## Reprise

I have emphasized certain tensions within Mrs. O's classes, but these came into view partly because I crouched there with one eye on the framework. The tensions I have discussed were not illusory, but my angle of vision brought them into focus. Another observer, with other matters in mind, might not have noticed these tensions. Mrs. O certainly did not notice them, and things went quite smoothly in her lessons. There was nothing rough or ungainly in the way she and her students managed. They were well used to each other, and to the class routines. They moved around easily within their math lessons. The various contrary elements of instruction that sent me reeling mentally did not disturb the surface of the class. On the contrary, students and teacher acted as though the threads of these lessons were nicely woven together. Aspects of instruction that seemed at odds analytically appeared to nicely coexist in practice.

What accounts for this smoothness? Can it be squared with the tensions that I have described within these classes? Part of the answer lies in the classroom discourse. Mrs. O never invited or permitted broad participation in mathematical discussion or explanation. She held most exchanges within a traditional recitation format. She initiated nearly every interaction, whether with the entire class or one student. The students' assigned role was to respond, not initiate. They complied, often eagerly. Mrs. O was eager for her students to learn; in return, most of her students seemed eager to please. And eager or not, compliance is easier than initiation, especially when so much of instruction is so predictable. Much of the discourse was very familiar to members of the class; often they gave the answers before Mrs. O asked the questions. So even though most of the class usually was participating in the discourse, they participated on a narrow track, in which she maintained control of direction, content, and pace.

The framework explicitly rejects this sort of teaching. It argues that children need to express and discuss their ideas, in order to deeply understand the material on which they are working (CSDE, 1985, pp. 14, 16). Yet the discourse in Mrs. O's class tended to discourage students from reflecting on mathematical ideas, or from sharing their puzzles with the class. There were few opportunities for students to initiate discussion, explore ideas, or even ask questions. Their attention was focused instead on successfully managing a prescribed, highly structured set of activities. This almost surely restricted the questions and ideas that could occur to students, for thought is created, not merely expressed, in social interactions. Even if the students' minds were nonetheless still privately full of bright ideas and puzzling mathematical problems, the discourse organization effectively barred them from the public arena of the class. Mrs. O employed a curriculum that sought to teach math for understanding, but she kept evidence about what students' understood from entering the classroom discourse. One reason that Mrs. O's class was so smooth was that so many possible sources of roughness were choked off at the source.

Another reason for Mrs. O's smooth lessons has to do with her knowledge of mathematics. Though she plainly wanted her students to understand this subject, her grasp of mathematics restricted her notion of mathematical understanding, and of what it took to produce it. She did not know mathematics deeply or extensively. She had taken one or two courses in college, and reported that she had liked them; but she had not pursued the subject further. Lacking deep knowledge, Mrs. O was simply unaware of much mathematical content and many ramifications of the material she taught. Many paths to understanding were not taken in her lessons—as for instance, in the Santas' letter example—but she seemed entirely unaware of them. Many misunderstandings or inventive ideas that her students might have had would have made no sense to Mrs. O, because her grip on mathematics was so modest. In these ways and many others, her relatively superficial knowledge of this subject insulated her from even a glimpse of many things she might have done to deepen students' understanding. Elements in her teaching that seemed contradictory to an observer therefore seemed entirely consistent to her, and could be handled with little trouble.

Additionally, however much mathematics she knew, Mrs. O knew it as a fixed body of truths, rather than as a particular way of framing and solving problems. Questioning, arguing, and explaining seemed quite foreign to her knowledge of this subject. Her assignment, she seemed to think, was to somehow make the fixed truths accessible to her students. Explaining them herself in words and pictures would have been one alternative, but she employed a curriculum that promised an easier way, that is, embody mathematical ideas and operations in concrete materials and physical activities. Mrs. O did not see mathematics as a source of puzzles, as a terrain for argument, or as a subject in which questioning and explanation were essential to learning and knowing—all ideas that are plainly featured in the framework (CSDE, 1985, pp. 13–14). *Math Their Way* did nothing to disturb her view on this matter. Lacking a sense of the importance of explanation, justification, and argument in mathematics, she simply slipped over many opportunities to elicit them, unaware that they existed.

So the many things that Mrs. O did not know about mathematics protected her from many uncertainties about teaching and learning math. Her relative ignorance made it difficult for her to learn from her very serious efforts to teach for understanding. Like many students, what she didn't know kept her from seeing how much more she could understand about mathematics. Her ignorance also kept her from imagining many different ways in which she might teach mathematics. These limitations on her knowledge meant that Mrs. O could teach for understanding, with little sense of how much remained to be understood, how much she might incompletely or naively understand, and how much might still remain to be taught. She is a thoughtful and committed teacher, but working as she did near the surface of this subject, many elements of understanding and many pedagogical possibilities remained invisible. Mathematically, she was on thin ice. Because she did not know it, she skated smoothly on with great confidence.

In a sense, then, the tensions that I observed were not there. They were real enough in my view, but they did not enter the public arena of the class. They lay beneath the surface of the class's work; indeed, they were kept there by the nature of that work. Mrs. O's modest grasp of mathematics, and her limited conception of mathematical understanding simply obliterated many potential sources of roughness in the lessons. And those constraints of the mind were given added social force in her close management of classroom discourse. Had Mrs. O known more math, and tried to construct a somewhat more open discourse, her class would not have run so smoothly. Some of the tensions that I noticed would have become audible and visible to the class. More confusion and misunderstanding would have surfaced. Things would have been rougher, potentially more fruitful, and vastly more difficult.

## Practice and Progress

Is Mrs. O's mathematical revolution a story of progress, or of confusion? Does it signal an advance for the new math framework, or a setback?

These are important questions, inevitable in ventures of this sort; but it may be unwise to sharply distinguish progress from confusion, at least when considering such broad and deep changes in instruction. After all, the teachers and students who try to carry out such change are historical beings. They cannot simply shed their old ideas and practices like a shabby coat, and slip on something new. Their inherited ideas and practices are what teachers and students know, even as they begin to know something else. Indeed, taken together those ideas and practices summarize them as practitioners. As they reach out to embrace or invent a new instruction, they reach with their old professional selves, including all the ideas and practices comprised therein. The past is their path to the future. Some sorts of mixed practice, and many confusions, therefore seem inevitable.

The point seems fundamental, yet it often goes unnoticed by those who promote change in teaching, as well as by many who study it. Larry Cuban's *How Teachers Taught* is a happy exception (1984). Cuban explained that " . . . many teachers constructed hybrids of particular Progressive practices grafted onto what they ordinarily did in classrooms" (L. Cuban, personal communication April 18, 1983). Cuban dubbed this approach to the adoption of innovations "conservative progressivism" (Cuban, 1984).

But these mixed practices affect the judgments that teachers and observers make about change in teaching. For instance, the changes in Mrs. O's teaching that seemed paradoxical to me seemed immense to her. Remember that when she began teaching four years ago, her math lessons were quite traditional. She ignored the mathematical knowledge and intuitions that children brought to school. She focused most work on computational arithmetic, and required much classroom drill. Mrs. O now sees her early teaching as unfortunately traditional, mechanical, and maladapted to children's learning. Indeed, her early math teaching was exactly the sort of thing that the framework criticized.

Mrs. O described the changes she has made as a revolution. I do not think that she was deluded. She was convinced that her classes had greatly improved. She contended that her students now understood and learned much more math than their predecessors had, a few years ago. She even asserts that this has been reflected in their achievement test scores. I have no direct evidence on these claims, but when I compared this class with others that I have seen, in which instruction consisted only of rote exercises in manipulating numbers, her claims seemed plausible. Many traditional teachers certainly would view her teaching as revolutionary.

Still, all revolutions preserve large elements of the old order as they invent new ones. One such element, noted above, was a conception of mathematics as a fixed body of knowledge. Another was a view of learning mathematics in which the aim was getting the right answers. I infer this partly from the teaching that I observed, and partly from several of her comments in our conversations. She said, for example, that math had not been a favorite subject in school. She had only learned to do well in math at college, and was still pleased with herself on this score, when reporting it to me years later. I asked her how she had learned to do and like math at such a late date, and she explained: " . . . I found that if I just didn't ask so many why's about things that it all started fitting into place . . . " (interview December 6, 1988). This suggests a rather traditional approach to learning mathematics. More important, it suggests that Mrs. O learned to do well at math by avoiding exactly the sort of questions that the framework associates with understanding mathematics. She said in another connection that her view of math has not changed since college. I concluded that whatever she has learned from workshops, new materials, and new policies, it did not include a new view of mathematics.

Another persistent element in her practice was "clinical teaching," that is, the California version of Hunter's Instructional Theory Into Practice (ITIP). Hunter and her followers advocate clearly structured lessons: Teachers are urged to be explicit about lesson objectives themselves, and to announce them clearly to students. They also are urged to pace and control lessons so that the intended content is covered, and to check that students are doing the work and getting the point, along the way. Though these ideas could be used in virtually any pedagogy, they have been almost entirely associated with a rigid, sonata-form of instruction, that is marked by close teacher control, brisk pacing, and highly structured recitations. The ITIP appears to have played an important part in Mrs. O's own education as a teacher, for on her account she learned about it while an undergraduate, and used it when she began teaching. However she also has been encouraged to persist: Both her principal and assistant principal are devotees of Hunter's method, and have vigorously promoted it among teachers in the school. This is not unusual, as ITIP has swept California schools in the past decade. Many principals now use it as a framework for evaluating teachers, and as a means of school improvement. Mrs. O's principal and the assistant principal praised her warmly, saying that she was a fine teacher with whom they saw eye to eye in matters of instruction.

I asked all three whether clinical teaching worked well with the framework. None saw any inconsistency. Indeed, all emphatically said that the two innovations were "complementary." Though that might be true in principle, it was not true in practice. As ITIP was realized in Mrs. O's class among many others, it cut across the grain of the framework. Like many other teachers, her enactment of clinical teaching rigidly limited discourse, closely controlled social interaction, focused the classroom on herself, and helped to hold instruction to relatively simple objectives.

As Mrs. O revolutionized her math teaching, then, she worked with quite conventional materials: A teacher-centered conception of instructional discourse; a rigid approach to classroom management; and a traditional conception of mathematical knowledge. Yet she found a way to make what seemed a profound change in her math teaching. One reason is that the vehicle for change did not directly collide with her inherited ideas and practices. *Math Their Way* focuses on materials and activities, not on mathematical knowledge and explanation of ideas. It allowed Mrs. O to change her math teaching in what seemed a radical fashion while building on those old practices. This teacher's past was present, even as she struggled to renounce and surpass it.

Mrs. O's past also affected her view of her accomplishments, as it does for all of us. I asked, in the Spring of 1989, where her math teaching stood. She thought that her revolution was over. Her teaching had changed definitively. She had arrived at the other shore. In response to further queries, Mrs. O evinced no sense that there were areas in her math teaching that needed improvement. Nor did she seem to want guidance about how well she was doing, or how far she had come.

There is an arresting contrast here. From an observer's perspective, especially one who had the new framework in mind, Mrs. O looks as though she may be near the beginning of growth toward a new practice of math teaching. She sees the matter quite differently: She has made the transition, and mastered a new practice.

Which angle is most appropriate, Mrs. O's or the observers? This is a terrific puzzle. One wants to honor this teacher, who has made a serious and sincere effort to change, and who has changed. One also wants to honor a policy that supports greater intelligence and humanity in mathematics instruction.

It is worth noticing that Mrs. O had only one perspective available. No one had asked how she saw her math teaching in light of the framework. She had been offered no opportunities to raise this query, let alone assistance in answering it. No one offered her another perspective on her teaching. If no other educators or officials in California had seen fit to put the question to her, and to help her to figure out answers, should we expect her to have asked and answered this difficult question all alone?

That seems unrealistic. If math teaching in California is as deficient as the framework and other critiques suggest, then most teachers would not be knowledgeable enough to raise many fruitful questions about their work in math by themselves, let alone answer them. We can see some evidence for this in Mrs. O's lessons. Their very smoothness quite effectively protected her from experiences that might have provoked uncertainty, conflict, and therefore deep questions. Even if such questions were somehow raised for Mrs. O and other teachers, the deficiencies in their practice, noted in many recent reports, would virtually guarantee that most of these teachers would not know enough to respond appropriately, on their own. How could teachers be expected to assess, unassisted, their own progress in inventing a new sort of instruction, if their math teaching is in the dismal state pictured in the policy statements demanding that new instruction?

Additionally, if teachers build on past practices as they change, then their view of how much they have accomplished will depend on where they start. Teachers who begin with very traditional practices would be likely to see modest changes as immense. What reformers might see as trivial, such teachers would estimate as a grand revolution, especially as they were just beginning to change. From a perspective still rooted mostly in a traditional practice, such initial changes would seem—and be—immense. That seemed to be Mrs. O's situation. She made what some observers might see as tiny and perhaps even misguided changes in her teaching. However, like other teachers who were taking a few first small steps away from conventional practice, for her they were giant steps. She would have to take many more steps, and make many more fundamental changes before she might see those early changes as modest.

So, if California teachers have only their subjective yardsticks with which to assess their progress, then it seems unreasonable to judge their work as though they had access to much more and better information. For it is teachers who must change in order to realize new instructional policies. Hence their judgment about what they have done, and what they still may have to do, ought to be given special weight. We might expect more from some teachers than others. Those who had a good deal of help in cultivating such judgment—that it, who were part of some active conversation about their work, in which a variety of questions about their practice were asked and answered, from a variety of perspectives—would have more resources for change than those who had been left alone to figure things out for themselves.

The same notion might be applied to policies like the new framework that seek to change instruction. We might expect only a little from those policies that try to improve instruction without improving teachers' capacity to judge the improvements and adjust their teaching accordingly, for such policies do little to augment teachers' resources for change. In Mrs. O's case, at least, thus far the framework has been this sort of policy. We might expect more from policies that help teachers to cultivate the capacity to judge their work from new perspectives, and that add to teachers' resources for change in other ways as well. The new instructional policy of which the framework is part has not done much of this for Mrs. O.

What would it take to make additional, helpful, and useful guidance available to teachers? What would it take to help teachers pay constructive attention to it? Neither query has been given much attention so far, either in efforts to change instruction or in efforts to understand such change. Yet without good answers to these questions, it is difficult to imagine how Mrs. O and most other teachers could make the changes that the framework seems to invite.

## Policy and Practice

Mrs. O's math classes suggest a paradox. This California policy seeks fundamental changes in learning and teaching. State policymakers have illuminated deficiencies in instruction and set out an ambitious program for improvement. Policy thus seems a chief agency for changing practice. Yet teachers are the chief agents for implementing any new instructional policy: Students will not learn a new mathematics unless teachers know it and teach it. The new policy seeks great change in knowledge, learning, and teaching, yet these are intimately held human constructions. They cannot be changed unless the people who teach and learn want to change, take an active part in changing, and have the resources to change. It is, after all, their conceptions of knowledge, and their approaches to learning and teaching that must be revamped.

Hence teachers are the most important agents of instructional policy (Cohen, 1988; Lipsky, 1980), but the state's new policy also asserts that teachers are the problem. It is, after all, their knowledge and skills that are deficient. If the new mathematics framework is correct, most California teachers know far too little mathematics, or hold their knowledge improperly, or both. Additionally, most do not know how to teach mathematics so that students can understand it. This suggests that teachers will be severely limited as agents of this policy: How much can practice improve if the chief agents of change are also the problem to be corrected?

This paradox would be trivial if fundamental changes in learning and teaching were easy to make. Yet even the new framework recognized that the new mathematics it proposes will be ". . . difficult to teach" (CSDE, 1985, p. 13). Researchers who have studied efforts to teach as the framework intends also report that it is difficult, often uncommonly so. Students cannot simply absorb a new body of knowledge. In order to understand these subjects, learners must acquire a new way of thinking about a body of knowledge, and must construct a new practice of acquiring it (Lampert, 1988). They must cultivate strategies of problem solving that seem relatively unusual and perhaps counter-intuitive (diSessa, 1983). They must learn to treat academic knowledge as something they construct, test, and explore, rather than as something they accept and accumulate (Cohen, 1988). Additionally, and in order to do all of the above, students must unlearn acquired knowledge of math or physics, whether they are second graders or college sophomores. Their extant knowledge may be naive, but it often works.

A few students can learn such things easily. Some even can pick them up more or less on their own. However, many able students have great difficulty in efforts to understand mathematics, or other academic subjects. They find the traditional and mechanical instruction that the framework rejects easier and more familiar than the innovative and challenging instruction that it proposes.

If such learning is difficult for students, should it be any less so for teachers? After all, in order to teach math as the new framework intends, most teachers would have to learn an entirely new version of the subject. To do so they would have to overcome all of the difficulties just sketched. For, as the framework says of students, teachers could not be expected to simply absorb a new "body" of knowledge. They would have to acquire a new way of thinking about mathematics, and a new approach to learning it. They would have to additionally cultivate strategies or problem solving that seem to be quite unusual. They would have to learn to treat mathematical knowledge as something that is constructed, tested, and explored, rather than as something they broadcast, and that students accept and accumulate. Finally, they would have to unlearn the mathematics they have known. Though mechanical and often naive, that knowledge is well settled, and has worked in their classes, sometimes for decades.

These are formidable tasks, even more so for teachers than for students. Teachers would have a much larger job of unlearning: After all, they know more of the old math, and their knowledge is much more established. Teachers also would have to learn a new practice of mathematics teaching, while learning the new mathematics and unlearning the old. That is a very tall order. Additionally, it is difficult to learn even rather simple things—like making an omelette—without making mistakes; but mistakes are a particular problem for teachers. For one thing teachers are in charge of their classes, and they hold authority partly in virtue of their superior knowledge. Could they learn a new mathematics and practice of mathematics teaching, with all the trial and error that would entail, while continuing to hold authority with students, parents, and others interested in education? For another, teachers are responsible for their students' learning. How can they exercise that responsibility if they are just learning the mathematics they are supposed to teach, and just learning how to teach it? American education does not have ready answers for these questions. However, there was no evidence that the framework authors, or educators in Mrs. O's vicinity, had even asked them. It is relatively easy for policymakers to propose dramatic changes in teaching and learning, but teachers must enact those changes. They must maintain their sense of responsibility for student's accomplishments, and the confidence of students, parents, and members of the community. Unfortunately, most schools offer teachers little room for learning, and little help in managing the problems that learning would provoke.

The new mathematics framework seemed to recognize some problems that students would have in learning new mathematics, but, from Mrs. O perspective, the state has not acted as though it recognized the problems of teachers' learning. Mrs. O certainly was not taught about the new mathematics in a way that took these difficulties into account. Instead, the CSDE taught her about the new math with the very pedagogy that it criticized in the old math. She was told to do something, like students in many traditional math classrooms. She was told that it was important. Brief explanations were offered, and a synopsis of what she was to learn was provided in a text. California education officials offered Mrs. O a standard dose of knowledge telling. The state acted as though it assumed that fundamental instructional reform would occur if teachers were told to do it. New goals were articulated, and exhortations to pursue them were issued. Some new materials were provided. Although the state exhorted teachers to devise a new pedagogy for their classes, it did so with an old pedagogy.

If, as the framework argues, it is implausible to expect students to understand math simply by being told, why is it any less implausible to expect teachers to learn a new math simply by being told? If students need a new instruction to learn to understand mathematics, would not teachers need a new instruction to learn to teach a new mathematics? Viewed in this light, it seems remarkable that Mrs. O made any progress at all.

What more might have been done, to support Mrs. O's efforts to change? What would have helped her to make more progress toward the sort of practice that the framework proposed? It is no answer to the question, but I note that no one in Mrs. O's vicinity seemed to be asking that question, let alone taking action based on some answers.

This new policy aspires to enormous changes in teaching and learning. It offers a bold and ambitious vision of mathematics instruction, a vision that took imagination to devise and courage to pursue. Yet this admirable policy does little to augment teachers' capacities to realize the new vision. For example, it offers rather modest incentives for change. I could detect few rewards for Mrs. O to push her teaching in the framework's direction—certainly no rewards that the state offered. The only apparent rewards were those that she might create for herself, or that her students might offer. Nor could I detect any penalties for nonimprovement, offered either by the state or her school or district.

Similar weaknesses can be observed in the supports and guidance for change. The new framework was barely announced in Mrs. O's school. She knew that it existed, but wasn't sure if she had ever read it. She did know that the principal had a copy. The new framework did bring a new text series, and Mrs. O knew about that. She knew that the text was supposed to be aligned with the framework. She had attended a publisher's workshop on the book, and said it had been informative. She had read the book, and the teachers' guide. Yet she used the new book only a little, preferring *Math Their Way*. The school and district leadership seemed to have thought *Math Their Way* was at least as well aligned with the framework as the new text series, and permitted its substitution in the primary grades.

Hence the changes in Mrs. O's practice were partly stimulated by the new policy, but she received little guidance and support from the policy, or from the state agencies that devised it. There was a little more guidance and support from her school and district: She was sent to a few summer workshops, and she secured some additional materials. However, when I observed Mrs. O's teaching there seemed to be little chance that she would be engaged in a continuing conversation about mathematics, and teaching and learning mathematics. Her district had identified a few mentor teachers on whom she could call for a bit of advice if she chose. There was no person or agency to help her to learn more mathematics, or to comment on her teaching in light of the framework, or to suggest and demonstrate possible changes in instruction, or to help her try them out. The new mathematics framework greatly expanded Mrs. O's obligations in mathematics teaching without much increasing her resources for improving instruction. Given the vast changes that the state has proposed, this is a crippling problem.

Mrs. O's classroom reveals many ambiguities, and, to my eye, certain deep confusions about teaching mathematics for understanding. She has been more successful in helping her students to learn a more complex mathematics than California has been in helping her to teach a more complex mathematics. From one angle this situation seems admirable: Mrs. O has had considerable discretion to change her teaching, and she has done so in ways that seem well-adapted to her school. Though I may call attention to the mixed quality of her teaching, her superiors celebrate her work. From another angle it seems problematic. If we take the framework's arguments seriously, then Mrs. O should be helped to struggle through to a more complex knowledge of mathematics, and a more complex practice of teaching mathematics. For if she cannot be helped to struggle through, how can she better help her students to do so? Some commentators on education have begun to appreciate how difficult it is for many students to achieve deep understanding of a subject and that appreciation is at least occasionally evident in the framework. There is less appreciation of how difficult it will be for teachers to learn a new practice of mathematics instruction.

## Notes

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