Experiencing School Mathematics

Revised and Expanded Edition

Traditional and Reform Approaches to Teaching and Their Impact on Student Learning

Jo Boaler

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Jo Boaler
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For Julie Boaler – my sister and friend.

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Foreword

Alan H. Schoenfeld
University of California, Berkeley

I first read Jo Boaler's book *Experiencing School Mathematics* soon after it was published by the Open University Press in 1997. It was hard to find the book in the United States, but colleagues told me the search was worth it: Boaler's description of two very different English schools and the effects of the kinds of instruction the students in them received was interesting, at times apparently paradoxical, and, to put it simply, important.

Boaler's research took place in England, and it is about English schools—but no matter. Save for a few typically British quirks (e.g., students refer to their teachers as *sir* and *miss*), Boaler could have been describing events and environments I have seen in hundreds of classrooms across the United States. My reaction on reading the book was not simply "her descriptions ring true," which they do; it was, "I know those classrooms. I know the forms of instruction, and I recognize the ways in which students are reacting to it." That is important because *Experiencing School Mathematics* has a lot to say to Americans interested in mathematics instruction. More about that in a moment. Let me say something about what was in the original book, as well as the revised and expanded American edition now in your hands.

Experiencing School Mathematics is a multifaceted book. For 3 years, Boaler examined the experiences of students and teachers in two English schools, exploring multiple aspects of school cultures and the students' mathematics learning. She spent time with the teachers, coming to know them as individuals and understanding their approaches to instruction. She conducted extensive lesson observations, to the point where she could provide rich, thick descriptions of the "lived experiences" of the students during instruction. She interviewed the students about a wide range of topics, obtaining information that would allow her to construct portraits of the students' knowledge, beliefs, and mathematical identities. These qualitative, anthropological observations were mixed with a wide range of quantitative studies—extensive analyses of student performance on standardized exams and on specially constructed tasks that focused on aspects of problem solving left untapped by the standardized exams. The result is integrated and coherent: Boaler's multiple sources of evidence and triangulation on the data provide a robust and well-documented body of findings.

The two schools Boaler studied had remarkably similar demographics—similar student bodies of comparable socioeconomic status (SES), educational background, and entering performance on standardized tests—but different philosophies. In many way, Amber Hill school was the embodiment of traditional instruction. It had dedicated and competent teachers, a well-specified curriculum (the English National Curriculum), and a coherent department-wide approach to instruction:

> All the mathematics teachers were well qualified mathematics specialists . . . [who] believed that the most efficient and effective way to teach mathematics was to impart knowledge of different mathematical procedures, using the chalkboard, and then get students to practice these procedures individually. The teachers believed that if they explained mathematical methods clearly, the students would gain an understanding of them. The teachers also believed that students needed to do a large number of similar exercises, because the act of repeating a procedure they had learned would make students remember it.

The school climate was focused on getting work done: Measures of time on task indicated that students were doing their work an astoundingly high percentage of the time. Most Americans would recognize the classrooms at Amber Hill as familiar: In one lesson Boaler describes, for example, students are marched rapidly through the factorization of polynomials such as $x^2 - 3x - 4$.

Phoenix Park school, apart from the demographics it shared with Amber Hill, was something else altogether. In British parlance, the mathematics department at Phoenix Park had a progressive approach—an approach that might seem almost a caricature of reform curriculum and practice in the United States. The curriculum, which had been devised by the teachers over time, was project based. Students were given complex tasks and a good deal of time to sort through them, sometimes individually, sometimes in small groups. For example, one of the projects was called *Volume 216*. Students were told that the volume of a shape was 216. They were then asked to go away and think about what the shape could be.

After they posed a challenge to the class, teachers would then work with individual students, tailoring the task to the students' needs and abilities. Some students, for example, might simply work on factors of 216 to find rectangular solids of that volume; others might engage in more complex explorations. Amber Hill was "tracked" (in British parlance, students were assigned to different sets), whereas Phoenix Park was not: The open-ended nature of the tasks allowed heterogeneously grouped students to engage in the tasks at their own level (with guidance from the teachers). Although instruction at Amber Hill was focused directly on preparation for the standardized assessments (The General Certificate of Secondary Education [GCSE], which was closely tied to the National Curriculum), with only 3 weeks in Years 9 and 10 devoted to explorations of open-ended tasks, the reverse was the case at Phoenix Park: Students worked almost exclusively on projects, until "just in time" test preparation prior to the GCSE. Moreover, given its open and relaxed environment, time on task at Phoenix Park was much lower than at Amber Hill.

The impact of these experiences on students' knowledge, beliefs, and mathematical identities is described at length in the book. Here I want to focus on two main findings, giving the headlines to induce you to read the how and why behind them.

The first is that, when all was said and done, the Phoenix Park students did better on mathematics assessments than did the Amber Hill students. Performance on the GCSE was a wash: Although Amber Hill students did better on purely procedural aspects of the exam, Phoenix Park students did better on the conceptual parts—indeed, on any parts of the exam that called for even minor variations from the precise skills the students at Amber Hill had learned. There was, of course, no contest on tests of applications or problem solving: Amber Hill students were unprepared for them, whereas Phoenix Park students did well. Boaler's book thus provides some of the first comparative evidence that students who receive project-based instruction that does not focus on skills learn more—and different—mathematics than students receiving traditional skills-based instruction.

The second is that there were radically different patterns of gender-related mathematics performance at the two schools. At Amber Hill, there were significant differences in performance, with boys outperforming girls. More than boys, girls tended to dislike mathematics and think that they were bad at it. At Phoenix Park, there were no significant differences in performance between boys and girls. Moreover, slightly more girls than boys at Phoenix Park (and many more girls at Phoenix Park than at Amber Hill) said they enjoyed math and were good at it. Boaler's explanation for this phenomenon is interesting and likely to be controversial. Read it for yourself.

So what does a study of two schools in England in the mid-1990s have to do with American mathematics education in the early 2000s? A great deal—here is why. First, the themes that Boaler studied are universals. Amber Hill is an archetype—a representative of a large equivalence class of schools that teach in the traditional manner. As noted earlier, I have lived in classrooms much like those Boaler describes: My analyses of the devastating consequences of procedure-oriented instruction in American classrooms (see e.g., my 1988 article, "When Good Teaching Leads to Bad Results: The Disasters of Well Taught Mathematics Classes") describe the same kind of respectful, focused, procedurally based instruction that Boaler describes. I was hardly the first: Max Wertheimer's (1945/1959) discussion of the "parallelogram problem" showed that similarly narrow instruction resulted in similarly bad consequences in mid-20th-century Germany. Likewise, as early as the 1930s (see Fawcett, 1938/1995), the literature offered descriptions of how students can become independent and productive mathematical thinkers when their classroom environments support the development of such dispositions and skills. However, comparisons between the two kinds of instruction have been implicit, rather than direct. Boaler's book offers a direct comparison. It provides rich, thick descriptions and state-of-the-art research triangulation. Moreover, the book has been updated and revised in at least three important ways. The world has moved forward, both empirically and theoretically, since Boaler wrote the first edition; readers will find a significantly expanded, up-to-date set of references. Readers will also find expanded discussions of what took place in the classrooms at Phoenix Park and Amber hill, as well as treatment of issues that emphasizes the meaningfulness of her findings to current educational debates in the United States.

Finally, I should point out that Boaler's findings, the first of their kind, are now being joined by others that document similar results. An extended set of studies conducted in the past half dozen years (Senk & Thompson, in press) now provides compelling across-the-boards evidence that it is not an either-or—that conceptually oriented instruction can result in students' learning both concepts and procedures. As Kilpatrick (in press) summarizes the results,

> Students studying from *Standards*-based curricula do as well as students studying from traditional curricula on standardized mathematics tests and other measures of traditional content. They score higher than those who have studied from traditional curricula on tests of newer content and processes highlighted in the *Standards* document. These results indicate that *Standards*-based curricula are working in classrooms in ways their designers intended for them to work. (p. 2)

But those are results. The compelling detail of Boaler's work, which is really what counts, is in the pages that follow. Read on.

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Preface

I was working in my native England, the year was 1996, and I had just spent 3 years studying mathematics teaching and learning in two schools that offered vastly different approaches to mathematics. I knew that I had uncovered some important and controversial facts about the impact of the 'traditional' and 'reform' approaches I had studied, and it was time to write them up in a way that was true to the data and helpful to other teachers. I wrote a book that was published in 1997, which was later named as the Outstanding Book of the Year for education in Britain. At that time, I had no idea of the events that would follow—events that brought me to work in the United States at Stanford University. Now it seems time to communicate the results of that study again and to communicate them somewhat differently. In doing so, I hope to help American and other non-British readers make sense of aspects of the English education system that I now know to be different from those in other countries and need some explanation. I also take this opportunity to add some new data and ideas from that study and beyond.

Having lived in the United States for the past few years, I have come to understand dimensions of American mathematics reform efforts and the politics that surround them firsthand. I have seen and experienced the ways in which the "math wars", and the strong claims that are made about the impact of traditional and reform instruction, have affected and continue to affect local teachers, schools, and districts with whom I work. Although I tried to make the first version of my book relevant to international readers, I am now in a much better position to understand and explain the issues to an American (indeed, international) audience. In many ways, Amber Hill was the archetype of the traditional school, and its analogues can be found across the United States. Phoenix Park was somewhat more idiosyncratic, but the mathematics approach shared many characteristics with U.S. schools using reform curriculum. In my visits to schools in the last 3 years, I see the issues that emerged from Amber Hill and Phoenix Park playing out in similar ways, and I am convinced that the lessons we may learn from those two schools apply to a number of different settings and countries. In this revised edition, I particularly highlight some of the implications for schooling in the United States.

One of the good things about writing a second edition of a book is that the author gets to address the shortcomings of the first version. I now realize that, although the previous book gave lots of data on student beliefs, understandings, and achievement in relation to different teaching approaches, it gave insufficient data on the teacher moves that played such an important role in the different learning environments. This is unfortunate partly because the lack of focus on teachers seemed to suggest that any implementation of the two broad approaches outlined in the book would have the same impact, which was not something I intended to convey. I have become particularly sensitive to this issue since working in California – an area where the math wars have taken hold and done enormous damage in schools. The fights over which curriculum should be adopted in schools have generally left teachers out of the equation and suggested that curriculum is everything. The first edition of my book may have perpetuated the simplistic notion that some approaches are *best*, which I hope to re-dress in this edition by giving some indication of the *particular* ways that teachers and students worked together to impact student learning within the different approaches. I would need a whole volume of books to communicate these in the detail they deserve, but I hope this book gives at least some indication of the importance of teachers' decisions and actions as they implement different approaches.

Another difference between this and the previous edition of this book is the use of American English rather than British English. In this edition, I use American terms wherever possible, changing British terms such as *local education authority* to their closest American equivalents (*school district*). I have not changed any of the actual quotes given by teachers and students, even when they use the most colorful of British expressions as those are important research data, but I have added brief translations in parentheses where appropriate. This edition also includes an updated set of references and connections with new literature and research.

The insights in this and the previous version of my book were made possible by the teachers and students at Amber Hill and Phoenix Park schools. I am profoundly grateful to teachers and students from both schools for the time they generously gave me and for their friendship. I

was also extremely fortunate to be working in the company of many great scholars at King's College, London, during my research study. A number of my colleagues generously gave time and guidance, particularly Paul Black, Dylan Wiliam, Mike Askew, and Stephen Ball. Paul was one of my PhD supervisors; his rigorous approach to data analysis and careful attention to the support of his students were inspirational to me and continue to guide my work.

In preparing the new edition of this book, I was helped by a number of my new American friends and colleagues. Mark Saul read the entire book and provided many insightful comments that pushed and clarified my thinking. I am extremely grateful to Mark for the time he generously gave; this second edition is better for his insights. Emily Shahan also read the entire book with a careful American eye, pointing out all the Britishisms I needed to change, which was extremely helpful. Deborah Loewenberg Ball has been my closest colleague since I arrived in the United States; her support and friendship, both intellectual and personal, are evident in many of the improved sections of this new edition. I would also like to extend a big thank you to Alan Schoenfeld, who encouraged me to write this new edition for his series and provided steady guidance throughout. Finally, but most important, I thank Colin Haysman, whose support—whether it be intellectual, personal, or gastronomical—is constant and unquestioning.

Welcome to this book. I hope that readers will find it interesting and informative, and that the experiences and reflections of the students at Phoenix Park and Amber Hill schools are useful in prompting further understandings of the relationships between mathematics teaching and learning.

—Jo Boaler

1

Introduction

The question of which approach we should use to teach mathematics in schools is one that has perplexed parents, teachers, mathematicians, and others for decades (Benezet, 1935a, 1935b, 1936). In some ways, it is incredible that opinions are so divided around this question (Becker & Jacob, 2000), but in others it is not. Both because teaching is a highly complex event, but also because we have lacked careful research on the impact of different approaches to mathematics teaching and learning. Part of the aim of this book is to go some way toward changing this by communicating the experiences of two groups of students—approximately 300 in all—who experienced vastly different approaches to mathematics teaching and learning. The two schools that are the focus of this book, and the teachers who willingly offered their classrooms as sites for analysis, provide an unusual opportunity for us to learn about the ways that students' experiences in classrooms impact the knowledge, beliefs, and understanding they develop. The stories told by the students in the pages that follow are important: They give detailed insights into the ways that mathematics *teaching* affects mathematics *learning*. Their ideas, combined with various data on their work, stand as rare testimony to the potential of different forms of mathematics teaching and learning.

Within mathematics education, there is an established concern that many people are unable to use the mathematics they learn in school in situations outside the classroom. In various research projects, individuals have been observed using mathematics in real-world situations such as street markets, factories, and shops. In these settings, school-learned mathematical methods and procedures are rarely used (Lave, Murtaugh,

& de la Rocha, 1984; Lave, 1988; Masingila, 1993; Nunes, Schliemann, & Carraher, 1993). Lave (1988) has used these research findings to challenge the traditional conception of mathematics as an abstract and powerful tool that is easily transferred from one situation to another. She proposed that knowledge, rather than being a free-standing, transferable entity, is shaped or constituted by the situation or context in which it is developed and used. Such ideas have received strong support in the fields of anthropology, psychology, and education and are now pivotal to emerging theories about human cognition. Indeed, Lave and others in the field of situated learning (Brown, Collins, & Duguid, 1989; Greeno & MMAP, 1998; Young, 1993) have been instrumental in raising awareness of the importance of the situation, the context, or the community of practice (Lave & Wenger, 1991) in which ideas are encountered for the capabilities that learners subsequently develop. One of the aims of this research study was to explore the notion of situated learning and investigate the experiences of students when they needed to transfer mathematics from one situation to another. I was interested to discover whether different teaching approaches would influence the *nature* of the knowledge that students developed and the ways that students approached new and different situations. To do this, I monitored the impact of the students' contrasting mathematical environments on the beliefs and understandings that students developed and the effectiveness of these in different situations, including the national school leaving examination as well as more applied and realistic tasks.

In designing my research, I was aware that previous studies had evaluated the experiences of students taught using contrasting approaches to mathematics, most of these demonstrating advantages of open or activity-based approaches for students' performance on tests (Athappilly, Smidchens, & Kofel, 1983; Keedy & Drmacich, 1994; Maher, 1991; Resnick, 1990; Sigurdson & Olson, 1992). However, there appeared to be little research that examined the nature and form of the teaching that contributed toward differential test achievement (Hiebert et al., 1997). My aim was not only to monitor the effectiveness of two schools' approaches, but to examine the intricate and complex ways in which the different approaches influenced students, including the influence of the curriculum used, the teaching decisions made, and the teacher–student interactions (Cohen, Raudenbush, & Ball, 2000; Stein, Smith, Henningsen, & Silver, 2000). To achieve this, I conducted in-depth, longitudinal studies of students taught using different curriculum approaches over 3 years, combining analyses of their achievement with data on their teaching and learning experiences.

Over recent years, there has been recognition that teaching occurs in particulars (Ball & Cohen, 1999), and that the effectiveness of any teaching and learning situation will depend on the actual students involved, the actual curriculum materials used, and the myriad of decisions that teachers make as they support student learning. Teachers have traditionally been offered general principles and strategies about education, abstracted from the particulars of specific teaching and learning events. Such abstract knowledge can, in certain circumstances, be extremely powerful, but it leaves to teachers the task of translating it into practical action in their own classroom (Black & Wiliam, 1998; Schwab, 1969). An exciting development of recent years is the awareness that actual records of teaching and learning—videos of classrooms, teacher reflections, student work, and other materials—are extremely powerful sites for learning (Ball & Cohen, 1999). Teachers and others are finding that analyses grounded in actual practice allow a kind of awareness and learning that has not previously been possible. I hope this book provides such an example of practice and that the details that are portrayed will serve as sites for learning. In offering a detailed case study of different teaching approaches, I am not providing evidence of hundreds of examples of the same approach, but details that will allow teachers and researchers to make their own decisions about the aspects from which they may generalize and from which they may learn.

The story that will be told in this book concerns the mathematics teaching and learning in two schools. But this is not just an account of different mathematics approaches; it is also about different educational systems, popularly characterized as *traditional* and *reform*; about tracked and mixed-ability teaching; about gender and learning styles; and about the ways these factors play out in the day-to-day experiences of students in classrooms. The messages that emerged from the two schools were varied and, at times, unexpected. My ability to tell the story and communicate the systematic differences between the teaching methods at the two schools derives from the clear and open ways in which students reported their experience of the learning process. The students portrayed in this book will take the reader some way toward the worlds of school mathematics as the students experienced them. Furthermore, the students' actions, reflections, and descriptions provide important insights into the influence of the teaching approaches they encountered on the understanding they developed. The fact that this research is focused on two schools raises questions about its generalizability, but I am happy for these questions to be raised and for the answers to be sought within the pages of this book.

In Chapters 1 and 2, I introduce the study, students, and research methods. Chapter 3 introduces readers to the two schools and mathematics departments, and it gives an overview of the mathematics teaching approaches. Through descriptions of lessons and extracts from interviews, the reader will learn about the main features of the two teaching approaches. Chapters 4 and 5 introduce readers to Amber Hill and Phoenix

Park schools in more depth, with descriptions of the characteristics of the two approaches that emerged as central, and students' responses to them. In these chapters the reader will begin to enter the worlds of school mathematics as students' experienced them, reading about the students' experiences and beliefs through extracts of lessons and interviews. Chapters 1 to 4 give the reader a sense of school mathematics at Amber Hill and Phoenix Park, the events that transpired in the classrooms of the two schools, the important teaching moments, and students' responses to them. Chapters 6 to 8 offer an exploration and analysis of the students' understanding of mathematics, their beliefs about mathematics, and the ways these were influenced by the different teaching approaches. Chapters 9 and 10 deal particularly with issues of equity. Chapter 9 considers the ways in which the two approaches impacted girls' opportunities to learn and the implications that the new insights from this study have for analyses of gender. Chapter 10 deals specifically with tracking and the ways in which it shaped students' learning and precluded the attainment of equity. Chapter 11 summarizes ad reflects on the main research results considering what the Amber Hill and Phoenix Park students' experiences, understandings, and mathematical beliefs tell us about the effectiveness of different curriculum programs, teaching practices, and classroom environments.

2

The Schools, Students, and Research Methods

In the chapters that follow, the schools and students are described in some depth. The purpose of this chapter is to give a brief introduction to the two research settings, the students within them, and the methods used to monitor and understand their experiences.

RESEARCH METHODS

To contrast two different mathematical approaches, I conducted 3-year ethnographic case studies (Eisenhart, 1988) of the mathematical environments in two schools. As part of these case studies, I performed a longitudinal cohort analysis of a year group of students in each school as they moved from ages 13 to 16. In England, this would be called Year 9 to Year 11, but in this book I refer to the grade levels as Years 8, 9, and 10 to make them equivalent to U.S. grade levels. In England, students stay together in age cohorts all through school, and they take mathematics in every year from ages 5 to 16. Beyond age 16, mathematics becomes optional. In Grades 1 through 10, students do not *choose* mathematics courses, and mathematics is not divided into algebra and geometry as it is in the United States. The students are taught mathematics as a whole, each year, and the content as decreed by the National Curriculum[1] is made up of four content strands: number, algebra, shape and space, and data handling. The

[1]This was introduced in 1988 and revised and reintroduced in 1991. The curriculum was written by a group of mathematics educators, administrators, and business people. It sets out mathematics content, but does not recommend or prescribe actual teaching materials.

curriculum also includes a strand called *Using and Applying Mathematics*, which includes mathematical processes such as generalizing, justifying, communicating, proving, and reasoning, which are intended to infuse the other four content strands. Because students stay in the same age cohorts, and usually the same mathematics classes, throughout secondary school, I was able to monitor the incoming Year 8 students over 3 years of school as they went from ages 13 to 16. At age 16, students take the same national examinations in every subject—called the General Certificate of Secondary Education (GCSE). The mathematics GCSE served as one helpful indicator of the mathematics students learned in the different approaches I followed. This examination is made up of a large collection of short questions. Unlike the large-scale assessments used in the United States, these are all open response—there are no multiple-choice questions. These examinations are graded nationally by examination officials (generally experienced teachers).

The case studies I conducted of the teaching and learning in the two schools included a variety of qualitative and quantitative methods. An overview of the research methods used is given in Appendix 1. I chose to combine these different research strategies partly because of a belief that qualitative and quantitative techniques are not only compatible, but complementary. I also used a number of different techniques in an attempt to represent what Ball (1995) terms the "mobile, complex, ad hoc, messy and fleeting qualities of lived experience" (p. 6). Ball (1995) and Miles (1982) both warn of the danger of reducing the complexity of experience and striving toward a theory that it "all makes sense" (Miles, 1982, p. 126). In analyzing the practices of two schools, I did not wish to provide a definitive explanation of events, but a way of thinking that raised issues and questions about various features of school life. To this end, my research design was governed by the need to view events from a number of different perspectives and conceptualize factors such as enjoyment and understanding in different ways.

To understand the students' experiences of mathematics, I observed approximately 100 one-hour lessons in each school, usually taking the role of a participant observer (Eisenhart, 1988; Kluckhohn, 1940). I interviewed 32 students in Year 9 and 44 students in Year 10. I analyzed comments elicited from students and teachers about classroom events (Beynon, 1985). I gave questionnaires to all of the students in my case study year groups each year. I interviewed teachers at the start and end of the research, and I collected an assortment of background documentation. These methods, particularly the lesson observations and student interviews, enabled me to develop an understanding of the students' experiences and begin to view the worlds of school mathematics from the students' perspectives (Hammersley, 1992). To locate the students' perspectives within a broad understanding of the two schools, I also spent time "hanging out" (Delamont, 1984) in the faculty rooms and corridors of the schools; I socialized with teachers, and I tried to develop a sense of the two schools in as many ways as possible.

In addition to these methods, I gave the students various assessments during the 3-year period. Most of these I designed myself, but I was also given permission to conduct a detailed analysis of the students' GCSE examination responses. The various assessment activities and questions I used during the 3 years involved individual and group work and written and practical work. All of the research methods employed within the study were used to inform each other in a continual process of interaction and reanalysis (Huberman & Crandall, 1982). Observation data were collected and analyzed using a grounded theory approach (Glaser & Strauss, 1967), and field notes and interviews were analyzed through a process of open coding (Strauss & Corbin, 1990). Table 2.1 gives an overview of the different methods employed. For more details on the methodology and methods, see Boaler (1996a).

THE STUDENTS INVOLVED

The overall aim of my research study was to monitor the experiences of a year group of students as they moved from Years 8 to 10, but time constraints meant that some of my research methods needed to be focused on particular groups of students within the two year groups. For example, my lesson observations, interviews, and applied assessments could not be conducted with all of the mathematics groups in each year because of the time required by these methods. At Amber Hill, the year group was divided into eight ability groups (Sets 1–8) who were all taught mathematics at the same time. In England, most mathematics departments teach students in ability groups, known as *sets*, with Set 1 students perceived to be the most able. The different sets are taught similar content, but the higher sets are generally taught at a faster pace and cover more difficult material. Trigonometry, for example, is usually only introduced to students in higher groups. The national mathematics examination that students took at age 16 was divided into three levels: foundation, intermediate, and higher. These have some common questions, but the higher paper has some more demanding questions and enables students to attain higher grades (A*, A, B, C, or fail). The intermediate paper gave access to the middle grades (C, D, E, or fail), and the foundation paper gave access to only the lower grades (D, E, F, G, or fail). The set in which students are placed has significant implications for their attainment some years later. Any grade in the examination from A to G is considered a pass, which means

TABLE 2.1
Research Methods Used in the Study

Time in Study		*Research Method*	*Students/Teachers Involved*
Year 8	Term 1	Interviews	Four teachers from Amber Hill. Three teachers from Phoenix Park
		Seven contextualized short test questions	All year group in both schools, n = 305
	Term 2	Lesson observations, one full week in each school	Approximately 25 lessons in each school
		Questionnaires (including open and closed questions)	All year group in both schools, n = 263
	Term 3	Applied architectural activity and related test	Half of four groups in each school, n = 104
Year 9	Term 1	Lesson observations	Approximately 10 lessons per school
		Long-term learning tests	Two groups in each school, n = 61
		Lesson observations, one full week in each school	Approximately 25 lessons in each school
	Term 3	Seven contextualized short test questions	All year group in both schools, n = 268
		Questionnaires (including open and closed questions)	Year group 7, 8, and 9 in both schools, n = 653
		Interviews	16 students each from Amber Hill and Phoenix Park
		Applied "design an apartment" activity and related test	Four groups in each school, n = 188
		Lesson observations	Approximately 10 lessons per school
Year 10	Term 1	Lesson observations, one full week in each school	Approximately 25 lessons per school
	Term 2	Interviews	24 students from Amber Hill, 20 from Phoenix Park
		Questionnaires (closed responses only)	All year group in both schools, n = 202
		Interviews	Three teachers from each school
		Lesson observations	Approximately five lessons per school
		Analysis of GCSE examination responses	All GCSE entrants in each school, n = 290

students have attained GCSE certification. However, advanced mathematics study at 16 and beyond generally requires an A, B, or C grade, as do a number of professional occupations. Amber Hill, the more traditional school, placed students into sets at age 13, and there were eight sets, from 1 to 8, with Set 1 being the highest set. The division of students into eight narrow bands meant that teachers could teach a narrow band of content. At the more progressive school, which I call Phoenix Park, the students were taught in mixed ability groups in Years 8, 9, and 10, until a few weeks before the national examination when they were placed in three groups—targeting each of the examination levels—and given more focused preparation for the national examination.

Because of time limits, I decided at an early point in my study to focus some of my data-collection methods on Sets 1 to 4 at Amber Hill. This decision was not made because I was particularly interested in "high-ability" students. The decision was made mainly because the department chair was most comfortable with me visiting these groups and partly because the students in Sets 1 to 4 demonstrated some interesting patterns of performance in the first applied assessment activity I gave them. Therefore, I decided that most, but not all, of my lesson observations for my case study year group would be of Sets 1 to 4, and only those students, who I came to know well, would take applied assessments. In my observations of other year groups at Amber Hill, I observed students in the full range of sets (1–8). At Phoenix Park school, there were five mixed-ability mathematics groups who were taught mathematics at different times of the day. This meant that I could watch up to three of my case study year group lessons in one visit to Phoenix Park and I did not need to focus my methods on particular groups. My lesson observations, interviews, and assessments involved all five groups. When I was not observing lessons with my case study year group, I watched lessons in other year groups. All other research methods, including questionnaires, short assessment questions, observations of other year groups, and GCSE analyses, were carried out with the full range of groups in each school.

INITIAL ENTRY MEASURES

At the beginning of my research study, when the students were just starting Year 8, I analyzed the results of tests designed by the National Foundation for Educational Research (NFER), which both schools had administered to all their students at the beginning of Year 8.[2] These were mathematics tests, focusing in particular on numeracy. NFER provides national results for these tests, so I was able to standardize the results of both schools. The results for these tests are given in Table 2.2.

There were no significant differences between the test results at the two schools. Seventy-five percent of Amber Hill students and 77% of Phoenix Park students were below the national average for the test. I also adminis-

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[2]Forty of the Amber Hill students did not take this test due to their arrival in the second week of school. Of these students 17, went into Sets 1–4, 23 went into Sets 5–8, so the students' attainment was spread across the Amber Hill range.

TABLE 2.2
Standardized NFER Scores
(mean 100, standard deviation 15)

		73 to 82	*82 to 91*	*91 to 100*	*100 to 109*	*109 to 118*	*118+*	*n*
%	AH	25	25	25	16	8	2	160
	PP	17	35	25	17	6	2	109

tered my own set of questions to the case study year groups at both schools. These were seven contextualized questions assessing various aspects of number work. The results for these questions are given in Tables 2.3 to 2.5. Grade 1 is the correct answer in each case.

There were no significant differences between the two schools on any of these questions. At the beginning of Year 8, the students therefore appeared to have reached similar levels of attainment measured on a broad range of mathematics questions. There were also no significant differences between the two schools regarding sex, ethnicity, or social class.

TABLE 2.3
Number Difference Problem in Two Contexts (%)

	Chocolate Splits		*Tug of War*	
Grade	*Amber Hill*	*Phoenix Park*	*Amber Hill*	*Phoenix Park*
1	49	41	49	43
2	53	54	44	47
3	8	5	5	1
4	1	1	3	1
n	195	110	195	110

TABLE 2.4
Number Group Problem in Two Contexts (%)

	Cutting Wood		*Fashion Workshop*	
Grade	*Amber Hill*	*Phoenix Park*	*Amber Hill*	*Phoenix Park*
1	58	55	57	60
2	19	28	23	23
3	17	13	11	12
4	6	4	9	6
n	195	110	195	110

TABLE 2.5
Addition of Fraction Problem in Three Contexts (%)

	Penalties		*Plants*		*Abstract*	
Grade	*Amber Hill*	*Phoenix Park*	*Amber Hill*	*Phoenix Park*	*Amber Hill*	*Phoenix Park*
1	3	6	2	4	1	1
2	3	1	3	2	7	7
3	93	92	94	91	81	83
4	1	1	2	4	10	9
n	195	110	195	110	195	110

3

An Introduction to Amber Hill and Phoenix Park Schools

Both Amber Hill and Phoenix Park schools lie in the heart of mainly White, working-class communities located on the outskirts of large cities. Both schools are surrounded by council-owned houses (similar to projects in the United States) where the majority of children live. Neither school is selective, and most parents choose the schools for their proximity to their houses. In an analysis of socioeconomic status (SES) derived from fathers' occupations, there were no significant differences between the year groups in the two schools. Amber Hill is a secondary school that begins with Year 6 when students are 11 years of age. There were about 200 students in the year group I followed: 47% of these were girls, 20% were from single-parent families, 68% were classified as working class, and 17% were from ethnic minorities. Phoenix Park is an upper school and the students start in Year 8 when they are 13 years of age. There were approximately 110 students in the year group that I followed: 42% of these were girls, 23% were from single-parent families, 79% were classified as working class, and 11% were from ethnic minorities. A comparison of the initial attainment of the students revealed that there were no significant differences between the students at the beginning of Year 8 (see chap. 2).

AMBER HILL SCHOOL

Amber Hill school is a mixed, comprehensive school, which means it is a local, public school with no entry requirements that accepts all local students of either sex. There were approximately 1,200 students between the ages of 11 through 18 at the school. It is located in the main working-class area of Fieldton, a large suburb of a major city. The majority of students who attend the school are White and working class, and the school is usually placed at or near to the bottom of the table of secondary schools that orders all schools in the local district by academic outcome.

The school is located in a quiet residential road overlooked by two high-rise apartment blocks in which many of my case study students lived. One of the first things I noticed when I began my research was the apparent respectability of the school. Walking into the reception area on my arrival, I was struck by the tranquillity of the arena. The reception was separated from the rest of the school by a set of heavy double doors. The floors were carpeted in a somber gray; a number of easy chairs had been placed by the secretary's window and a small tray of flowers sat above them. The walls displayed photographs from sports days and school trips, awards that students had received, and Amber Hill emblems and coats of arms. Icons of traditionalism were located throughout the reception area, presenting strong messages about the way in which the school was intended to be perceived.

The principal of Amber Hill was a particularly important and influential figure. John Patram was the archetype of the "authoritarian head" (Ball 1987, p. 109), particularly in his attitude toward opposing views, which were "avoided, disabled or simply ignored" (p. 109). The mathematics teachers reported that he imposed decisions on teachers after consultations, which he ignored. John Patram had an austere appearance—he was always dressed in a dark suit and wore a solemn expression. At break times, he wandered the corridors shouting at students; the teachers seemed as unwilling to bump into him as the students. He rarely visited the faculty room or socialized with teachers.

Partly as a result of the principal's influence and power, Amber Hill was unusually orderly and controlled. Students generally did as they were told, their behavior governed by numerous enforced rules and a general school ethos that induced obedience and conformity. All students were required to wear a school uniform, which the vast majority of students wore exactly as the regulations required. The annual school report that teachers sent home to parents required the teachers to give the students a grade on their "co-operation" and their "wearing of school uniform." The head clearly wanted to present the school as academic and respectable, and he was successful in this aim at least in terms of the general façade. Visitors walking around the corridors would see unusually quiet and calm classrooms, with students sitting in rows or small groups usually watching the board. When students were unhappy in lessons, they tended to withdraw instead of being disruptive. The corridors were mainly quiet, and at break times the students walked in an orderly fashion between lessons. The students' lives at Amber Hill were, in many ways, structured, disciplined, and controlled.

The Amber Hill Teachers

There were 70 teachers at Amber Hill who were quite polarized in terms of age. A large number of the teachers had been at the school for 10 years or more while a similar proportion were in their 20s and had been teaching for less than 3 years. The teachers appeared to mix well, although few of them spent their noncontact time in the teachers room as I and the mathematics department did. The remainder chose to stay within their subject domains. This tendency is indicative of teachers who have been socialized into strong subject loyalties (Bernstein, 1971).

The faculty room was split into two main rooms—one for smokers and the other for nonsmokers. The main core of the mathematics department always sat in the smoking section apparently because two of them smoked. The faculty room did not seem to be a particularly social place: Few teachers visited it at break times apart from the mathematics department who taught close by. Five of the mathematics department had commandeered their own comfortable chairs in which they always sat. The smoking section of the faculty room tended to be a lively arena in which complaints about various students' behavior in mathematics lessons or reports about the amount of work a student had completed would be banded about and discussed.

The mathematics department had nine members, including one teacher who worked half time and one who taught information technology most of the time. Seven of the department had been at the school for between 8 and 18 years, two for between 3 and 4 years. The department chair, Tim Langdon, was in his mid-30s and had been at the school for 4 years at the start of my research. Tim liked the curriculum the school used—the Secondary Mathematics Project (SMP) and had been involved in its choice (schools, rather than districts, choose curriculum in England). Tim regarded this to be an innovative curriculum, and the new publications that SMP issued from time to time made him feel that he was keeping abreast of the latest developments in mathematics education. Tim was also vocal in his support of the strand of the national curriculum that addressed mathematical processes. He thought that open-ended activities played an important role and encouraged other teachers to use them albeit on an infrequent basis. Tim was always friendly and amiable. He was also extremely conscientious and hard working; he would go to any lengths to help me with my research by, for example, organizing interviews, planning which lessons I could see, and sending me information and schedules.

The other teacher who helped me a great deal was Hilary Neville. Hilary was a mathematics teacher and the head of grade level for my case study cohort and so had a senior position in the school. Hilary was a forceful, efficient, and extremely competent woman in her 40s, who was both friendly and assertive with teachers and students. She was also both committed and hard working and obviously cared a great deal about the students. I became friends with both Hilary and Tim during my research and maintained contact with them after my study was completed.

Edward Losely was also an important figure in the department. At the start of my research, he was a newly qualified teacher of approximately 25 years of age. He was always grinning and joking with various teachers and helped to organize the student soccer and cricket teams. Edward was quite large and athletic looking and clearly enjoyed being "one of the lads." This extended to his lessons when he was often joking with boys, often referring to beer, pubs, football, and cricket in the examples he chose to describe mathematical situations. At break times, Tim, Hilary, and Edward would often sit and chat about mathematics lessons and students' behavior.

The rest of the teachers in the mathematics department were between ages 40 and 60 and shared the belief that SMP was an innovative curriculum. The teachers had concerns about individual students' mathematical knowledge and understanding, but they did not reveal any reservations about the SMP curriculum. All of the teachers complained to Tim about having to do investigational work and open-ended tasks, but they did believe in the occasional use of these activities.

All the mathematics teachers were well-qualified specialists, and all of them, Tim included, believed that the most efficient and effective way to teach mathematics was to impart knowledge of different mathematical procedures, using the chalkboard, and then get students to practice these procedures individually. The teachers believed that if they explained mathematical methods clearly, the students would gain an understanding of them. The teachers also believed that students needed to do a large number of similar exercises because the act of repeating a procedure they had learned would make students remember it. The teachers' belief in this didactic model of teaching meant that their main concern as teachers was to cover all of the necessary mathematical content:

> We've all done maths, so they've got the biggest resource standing in front of the class. And it's superb being able to—you've got the national curriculum basically and if you cover the national curriculum you're doing your job. (Edward Losely)

Mathematics Teaching at Amber Hill

Amber Hill used the SMP scheme throughout Years 6 to 10. In Years 6 and 7, the students worked through individualized SMP booklets, whereas in Years 8 to 10, they moved onto more formal textbooks. Students spent the first term of Year 6 in mixed-ability classes and then were grouped by perceived ability into eight mathematics sets. The allocation of students to sets was based on the results of NFER tests taken at the beginning of Year 6 and work completed in the first term of Year 6. In Years 6 and 7, students worked through the individualized booklets, at their own pace, with little or no whole-class teaching. In Year 8, they moved to a more formal system of textbooks and class teaching. There was no departmental policy about the way in which classes should work in Years 8 through 10, but all the teachers adopted the same pedagogical approach. They explained methods from the chalkboard at the front of the class for the first 15 to 20 minutes of each lesson; they then gave the students questions to work through from their SMP textbooks. This model of mathematics teaching has predominated in England for many years. It is likely that the teachers learned this approach from their own experiences as school students in a process that Lortie (1975) has referred to as "apprenticeship of observation" (p. 65). Most of the teachers questioned students while lecturing from the chalkboard. The students worked through textbooks in every lesson in Years 8 to 10, apart from 3 weeks of Years 9 and 10 when they completed an investigation or open-ended task. The distinct separation of the process and content areas of mathematics maintained within Amber Hill's approach is what Blum and Niss (1991) referred to as the "separation approach" (p. 60) common in many schools.

Most students sat in pairs in mathematics lessons, but they would work alone, usually stopping to check with their partner that they had the same answer at the end of each question. Teachers did not object to students talking quietly as they worked. All mathematics lessons were 1 hour long.

JB: What do you do in a typical math lesson?

J: Well sir usually goes over the work we have to do before we do it. So he'll write on the board what we have to do and explain the questions and that and the rules, the basics of what we have to do in the work and then he'll tell us to get on with it.

JB: From books?

J: Yeah from books and if we need help he'll come along and help us.

JB: And how long does he talk from the board and how long do you work from books?

J: About half a lesson. (John, AH, Year 9, Set 1)

The students worked from textbooks in each and every lesson. When they completed a chapter, they would do the textbook review, which assessed the work in the chapter:

A: It's always out of textbooks innit?

G: Yeah, we do a chapter, then we do a review and it's like that over and over again. (Alan & Gary, AH, Year 10, Set 3)

Lessons at Amber Hill were unusually ordered and controlled. Students were well behaved, and it was rare to see teachers invoke any disciplinary procedures against students. When the teachers talked from the front of the room, the students sat in silence listening to them, watching the board, and writing down what they were told. Students worked quietly through their exercises and confined any misbehavior to chatting with their partners. In lesson observations, I was repeatedly impressed by the work rate of the students. In a small quantitative assessment of their "time on task" (Peterson & Swing, 1982), I recorded the number of students who were working 10 minutes into, half way through, and 10 minutes before the end of each lesson. In a study of 10 representative lessons, each with approximately 30 students, 100%, 99%, and 92% of the students appeared to be working at these three respective times. The first of these figures was particularly high because, at this early point in lessons, the students were always watching the teachers work through examples on the board.

The Amber Hill students wanted to do well in mathematics and believed it to be an extremely important subject. This motivation, combined with their compliant behavior, meant that the teachers usually had captive audiences in lessons, with students who were willing to do whatever the teachers told them. The Amber Hill mathematics teachers developed good relationships with students. All of the teachers were friendly, and the students reported that they found them approachable and helpful.

PHOENIX PARK SCHOOL

Phoenix Park is also a local "comprehensive" public school with no entry requirements and students of both sexes attending. It is located on the edge of Avadon, a prosperous town with a large middle-class element. There were approximately 600 students at the school between the ages of 13 and 18. Like Amber Hill, most of the students at the school were White and working class. The majority of Phoenix Park students lived on one of three local housing estates (like U.S. projects), one of which was infamous for its links with "joy riding" and drug-related crimes. The school is situated in an industrial area, and a large proportion of the parents used to work in the local factories before widespread layoffs took place. The juxtaposition of this working-class school next to the affluent, middle-class city of Avadon made it somewhat distinctive in the locale. It was also distinct because of a long tradition of progressive education, placing particular emphasis on self-reliance and independence. Most of the parents who chose to send their children to Phoenix Park did so because they lived in the immediate vicinity of the school, rather than because of school philosophy or practice. In a school survey of 50 parents conducted in 1987, 44 parents said that their children lived within a 20-minute walk of the school. A few parents chose Phoenix Park because their children had special educational needs, which were given high priority in the school; a few chose the school because of its relaxed atmosphere. This contrasted with the more pressured and academic environments of the other schools in and around Avadon. Phoenix Park, like Amber Hill, was usually placed at or near the bottom of the local district table of schools ordered by academic achievement.

Phoenix Park school had an attractive campus feel. The atmosphere was unusually calm—described in a newspaper article on the school as *peaceful*. Students walked slowly around the school, and there was a noticeable absence of students running, screaming, or shouting. This was not because of school rules; it seemed to be a product of the school's overall ambiance. I mentioned this to one of the mathematics teachers one day and she agreed, saying that she did not think she had ever heard anybody shout—teacher or student. She added that this was particularly evident at break times in the hall: "The students are all so orderly, but no-one ever tells them to be" (Rosie Thomas).

Phoenix Park school maintained a number of distinctive qualities, most of which derived from its commitment to progressivism. In lessons, many of the subject departments used a project-based, problem-solving approach with little, if any, recourse to textbooks. Students were taught all subjects in mixed-ability groups. Phoenix Park students did not wear school uniforms. Most students wore fashionable but inexpensive clothes such as jeans, with trainers or boots, and shirts or t-shirts worn loosely outside.

A central part of the school's approach involved the development of independence among students. The students were encouraged to act responsibly—not because of school rules, but because they could see a reason to act in this way. In mathematics lessons, the teachers allowed the students to work on their own, unsupervised, in separate rooms as the students were expected to be responsible for their own learning.

> You've got a lot of freedom—it's not really like a school. The teachers don't treat you like kids. (Year 10 student, quoted in a school publication)

The school had a thriving special educational needs department, which it maintained throughout the late 1980s and early 1990s when many schools drastically reduced the number of teachers working within special educational needs. The school also had a commitment to equality of opportunity, which extended well beyond written policy documents.

Fletcher, Caron, and Williams (1985) describe the trouble experienced by schools that attempt to be progressive in their dealings with parents. Part of the freedom that Phoenix Park enjoyed in this regard seemed to be due to the working-class composition of the school and the presence of parents who were less inclined to challenge the authority of teachers. In the year after my 3-year research study, the school had an influx of middle-class parents who quickly put pressure on the teachers at the school to return to more traditional methods of schooling, including ability grouping and textbook teaching. This prompted a sequence of events that I describe at the end of this book. When I tell teachers in the United States about Phoenix Park, they often assume it is a private or charter school, but it is neither. Phoenix Park is a local, public school that, through a series of circumstances, developed a school-wide orientation toward progressive education.

The Phoenix Park Teachers

The teachers at Phoenix Park were relatively young, with approximately 30% of the teachers in their 20s, 30% in their 30s, and 30% in their 40s. Interactions between teachers were almost always casual and jovial. In my visits to the faculty room at Phoenix Park, I was always struck by its relaxed and cheerful atmosphere. Teachers did not seem to spend their break times complaining about workload, running around organizing detentions, or worrying about administration. Nor did they sit in separate subject departments talking about students who were or were not working. Instead, break and lunch times seemed to be social occasions in which teachers from different departments interacted and joked with each other.

The teachers at Phoenix Park were casually dressed. One day, one of the more senior members of the faculty was wearing a t-shirt with the name of a rock band on it, which prompted one of the other teachers to say, "one of the very nice things about this school is you can express yourself through your clothing!" The head teacher at Phoenix Park, Paul Mardon, did not seem distinct from other members of the faculty apart from the fact that he always wore a tie. He spent his lunch times wandering around the school grounds chatting to students; he knew all of the students by name, and they seemed comfortable in his presence.

When I began my research at Phoenix Park, the mathematics department was run by Sheila Rideout, who had a clear vision about the way mathematics should be taught. Sheila devised the mathematics approach at Phoenix Park, in conjunction with a working group of teachers adopting similar approaches in other schools. Sheila left Phoenix Park in the first year of my research, and her job as mathematics coordinator was taken over by Martin Collins, previously her deputy. A newly qualified teacher, Rosie Thomas, was appointed to the department to restore numbers. For the rest of my research, the department was made up of three and a *half* mathematics teachers. Martin, Rosie, and Jim all worked full-time in the mathematics department, and Barbara had a half-time contract at the school.

Martin Collins, the mathematics coordinator, was in his mid-30s. He had a mathematics degree and was well informed about developments in mathematics education. Martin was generally very laid back about everything, including teaching mathematics and running the department. He was not an active leader and was, in many ways, the complete opposite of Sheila. He was in favor of an open approach to teaching, but he had doubts about the effectiveness of the approach they used at the school.

Jim Cresswell was unusual, particularly for a teacher of mathematics. He was in his early 30s, he had an Oxbridge degree in engineering, and he was studying Chinese at degree level in his spare time. Jim used to be a community youth worker and was a practicing Quaker. He always dressed extremely casually, usually in faded jeans, a sweatshirt, and, in winter, a woolly hat. He had very short hair and an unshaven look. In the faculty room, he was often reading books about philosophy and politics. In the classroom, Jim treated the students as if they were adults; he rarely reprimanded them, and when students misbehaved he had conversations with them about the inconsiderateness of their behavior.

Rosie Thomas was a newly qualified teacher at the start of my research. She was in her early 20s, had a mathematics degree, and was enthusiastic about the school's approach and about teaching in general. She often chatted to students about mathematical and nonmathematical issues, and she was generally liked by students. Rosie quickly became involved in the life of the school, and she seemed to be a highly committed teacher.

Mathematics Teaching at Phoenix Park

Many of the progressive principles that underscored the whole-school philosophy of Phoenix Park were represented in the mathematics approach, which made it extremely unusual. From the beginning of Year 8 to

Christmas of Year 10, the students worked on open-ended projects in every lesson. During this time, the students were taught in mixed-ability groups. Projects usually lasted for about 3 weeks. The teachers introduced students to a project or theme, which the students explored using their own ideas and mathematical knowledge. The projects were usually extremely open, amounting to little more than a challenging statement.

T: The projects that we were set, we were actually given a title in the first . . . like what we had to do . . . but then after that you could decide how far you wanted to do it. (Tina, PP, Year 10, RT)

One of the projects was called *volume 216*. In this project, the students were told that the volume of a shape was 216. They were then asked to go away and think about what the shape could be. Students were expected to extend their work and pursue questions and interests related to this theme. Sometimes teachers taught the students some mathematical content they thought might be needed before the start of an activity. More commonly, teachers would introduce methods to individuals or small groups when they encountered a need for them within the particular project on which they were working.

S: We're usually set a task first and we're taught the skills needed to do the task, and then we get on with the task and we ask the teacher for help.
P: Or you're just set the task and then you go about it in . . . you explore the different things, and they help you in doing that . . . so you sort of . . . so different skills are sort of tailored to different tasks.
JB: And do you all do the same thing?
P: You're all given the same task, but how you go about it, how you do it and what level you do it at, changes, doesn't it? (Simon and Philip, PP, Year 10, JC)

The students were given an unusual degree of choice in mathematics lessons. When projects were introduced to them, they were usually given a few ideas to choose between; when they were working on their projects, they were required to decide the nature and direction of their work. Sometimes the different projects varied in difficulty, and the teachers guided students toward projects they thought were suited to their capabilities.

T: You get a choice.
JB: A choice between . . . ?

T: A couple of things, you choose what you want to do and you carry on with that and then you start another, different one.
JB: So you're not all doing the same thing at the same time?
Both: No.
JB: And can you do what you want in the activity, or is it all set out for you?
L: You can do what you want really.
T: Sometimes it's set out, but you can take it further. (Tanya and Laura, PP, Year 10, MC)

The pedagogy of Phoenix Park would be ideally described in Bernstein's terms as *invisible* because:

- the teachers had implicit rather than explicit control over students,
- the teachers arranged the *context* in which students explored work,
- students had wide powers over the selection and structure of their work and movements around the school,
- there was reduced emphasis upon the transmission of knowledge, and
- the criteria for evaluating students were multiple and diffuse. (Bernstein, 1975, p. 116)

The scheme of work used by the mathematics department looked incredibly sparse. Each academic year was split up into four or five topic areas. Within each area, the scheme of work gave a number of written objectives, a range of projects or investigations, and a list of national curriculum attainment targets. For example, in Year 8, the students were introduced to five topics: squares and cubes, connections and change, counting, geometry, and position and place. At departmental meetings, the teachers discussed the activities that they were about to use and any modifications they were intending to make, but there was little written documentation of the work. Some of the activities were written out on pieces of paper that were photocopied for the students, whereas others were written on the board at the beginning of lessons.

The mathematics department had a relaxed approach to both the national curriculum and assessment of work. Their scheme of work was cross-referenced to the national curriculum attainment targets, but had no finer level of detail than this. When the teachers assessed the students' projects, they wrote comments, describing what they considered to be good or bad about the work and ways in which students could improve the work. The teachers did not give grades except at the end of each year. Much of the teachers' assessment of the students was formative and took place as teachers walked around and interacted with students during lessons. Most groups were taught by the same teacher as they moved up the school unless a teacher left. The teachers at Phoenix Park tried to give a broad, holistic picture of students' achievement on particular projects; this stood in contrast to the marks or percentages the Amber Hill students frequently received for their answers to textbook questions.

At Phoenix Park, the teachers gave out projects and left students to develop them and use their own ideas. The teachers were available to help students, but the students could not rely on this help because there was only one teacher in each room. Students were encouraged to work together and help each other as part of their work, and most students did this.

The students at Phoenix Park learned mathematics through the use of open-ended projects until January of Year 10. At this time, they started examination preparation. The projects were abandoned, and students were introduced to any formal methods and notations they had not met. The students were grouped according to the examination they were entered for—foundation, intermediate, or higher. The teachers used the chalkboard more frequently to explain procedures, and the students practiced procedures within textbook questions, worksheets, and past examination questions. The students found this system of learning mathematics very different from the one to which they had, by then, become accustomed.

Prior to joining Phoenix Park in Year 8, all the students had attended middle schools that used the SMP scheme. This meant that in Years 6 and 7 students at both schools had learned mathematics using the same SMP 11-16 booklets. In Year 8, the students at the two schools then embarked on very different mathematical pathways. At Amber Hill, they moved to a more formal system of textbooks and class teaching. At Phoenix Park, the students abandoned set texts and moved to an extremely open, project-based approach. The chapters that follow describe the experiences of the students at the two schools over the next 3 years, describing the impact of these very different teaching methods on the students' development of mathematical understanding.

4

Amber Hill Mathematics: Experiences and Reflections

Over the years I spent studying Amber Hill's mathematics department, I recorded hundreds of observations, interviewed many different students (some successful, some not), and analyzed hundreds of responses to questionnaires and assessments. In presenting this chapter, which is intended to summarize the approach at Amber Hill, I have had to draw from the broader data set and make many choices along the way (Peshkin, 2001). It is likely that other researchers would have made other choices, but part of the ethical responsibility I maintain as a researcher is to portray data honestly and openly. I have therefore chosen data carefully, in consultation with others who read the data, to be representative and not distort the events that transpired at Amber Hill school. I triangulated data so that all of the conjectures and conclusions that follow are supported by at least two, usually three or more, data sources. Other researchers may have recorded or chosen different examples and noticed different events in the classrooms, but I am confident that they would have communicated a similar sense of the teaching and learning at Amber Hill. When the school underwent an official inspection as part of the National inspection service that all schools go through in England, the report raised many of the same issues that I highlight in the pages that follow.

In choosing data to represent the approach at Amber Hill, I have not only aimed for representativeness, I have also chosen data that seemed particularly important given the achievement results that transpired at the two schools. Thus, the meaning readers may draw from this work will probably increase in a cumulative manner as the different chapters unfold, as the students' reflections are heard, and as the achievement results are analyzed. For now it is probably important to note that the features of Amber Hill's teaching that I highlight are those that seemed particularly influential and had the greatest impact on the perceptions and understandings that students formed. In the first part of the chapter, I give an overview of the teaching approach at the school, drawing from various forms of data. In the second, I represent the students' responses to and ideas about school mathematics.

AMBER HILL TEACHING AND LEARNING

Structured Questions

At Amber Hill, the students worked from textbooks in each and every lesson in Years 8 to 10, apart from 3 weeks of Years 9 and 10 when they completed an open-ended coursework project. This meant that the vast majority of the students' experiences of mathematics involved short, procedural (Hiebert, 1986), and closed questions. Some more open questions did feature at the end of exercises, but when students encountered these questions the teachers would normally close them down. Doyle (1988) asserts that some teachers avoid classroom conflicts by "redefining or simplifying task demands" and "softening accountability to reduced risk" (p. 174). The Amber Hill teachers achieved this by breaking questions into small, atomistic parts and guiding students through any mathematical decision making. Some teachers isolated the more demanding questions in the chapters and put them up on the board prior to lessons. In other lessons, teachers broke the problems down for students in one-to-one situations or with the whole class when a problem caused difficulty. They would generally do this using what Doyle and Carter (1984) have referred to as "extensive teacher prompting" (p. 137). The teachers' motivation for this behavior was clear—they wanted to help the students and give them positive learning experiences. The following is a typical example taken from my field notes of a Year 8, Set 5 lesson with Tim Langdon.

> Tim announces that he is going to put the problem from the end of the chapter on the board. He draws:

Blagdon	0730	a	b
Westerfield	c	1045	d
Scaly Bridge	0845	1120	1535
Laughton	0935	e	f
New Harbour	g	h	1640

> *(This is intended to represent a train schedule, the names on the left are invented place names).*

> Tim then comes over to me and says, "this is the classic problem with SMP, it gets them working down columns linearly or across and then suddenly there's a massive jump to this." "And they can't do it?" I ask. "Well they can if you do this (he strokes my arm) and say 'come on you can do it, you can do it, do this bit and then do that bit.' " After Tim has put the problem on the board, he gets all of the students to listen and then asks Gary, who Tim knows has worked out (f), to explain how he got his answer. Gary mumbles, "you can see it takes one hour, no, 50 minutes to go from Scaly Bridge to Laughton so I done that, I added that onto Scaly Bridge and it come out 1625." While Gary is talking, the other students look distracted and do not appear to be listening to him. Tim says, "good, which letter shall we work out next?" Tracey offers (e). "Come on then Tracey," says Tim. "I'm not doing it," Tracey says. Tim then asks, "Well how long does it take to get from Scaly Bridge to Laughton?" "No idea," Tracey says. "Come on we've just heard how long!" Someone else calls out "50 minutes." Tracey repeats this. "OK," says Tim, sounding exasperated, "so what is 1120 plus 50 minutes?" "Dunno," says Tracey. Then "Oh, hold on, it's 1210." "See you can do it," says Tim. "Did I get it right?" Tracey asks with surprise. Tim says that she did. "Oh cocker," says Tracey, pleased.
>
> Tim moves through the problem asking different students similar small questions each time: "Well how long is it from here to here?" (pointing to two times). Tim asks Michael to do one of the letters. Michael says, "I can't do it." So Tim leads him through it: "How far did Leo say it was from here to here?" Michael trawls in his memory for the time, rather than trying to interpret the table. He gives the right answer. "So how far is it from here to here?" Tim continues. Eventually Michael gets to the answer, and Tim says, "wonderful, I thought you said you couldn't do it, have some confidence!" Tim continues with different students until all of the questions are completed. None of the students, even the last ones asked, attempt to get the answers without Tim leading them through the problem step by step (Year 9, Set 5, Tim Langdon).

When students encountered difficulty answering their questions at Amber Hill, the teachers would generally provide additional structure, creating more focused environments (Walker & Adelman, 1975). They did this by combining high-definition questions that had one correct answer, with a closed sequencing of content, moving in "tight, logical steps between one item and the next" (p. 47).

This is not unusual for mathematics teachers. Schoenfeld (1988) and Doyle (1988) both claim that mathematics teachers commonly provide students with detailed structure to help them solve problems. Other research studies, such as Barnes et al. (1969), have also suggested that the tendency of mathematics teachers to ask closed questions with short factual answers that do not require any interpretation or reasoning is not unusual. The predominance of the teachers' tendency to redefine questions and narrow their scope was not only evident in relation to questions, which were open, but it was also a more pervasive tendency that seemed to form the basis of mathematics instruction. In the majority of lessons I watched, the teachers would respond to the students' inability to answer questions by offering them a multiple-choice question, with one of two correct answers (e.g., "Well, is it 4 or 5?"). The students would select an answer and if this was right, the teachers moved on, "So is it the length or the width?", and so it proceeded. If students selected the wrong answer, the teachers would repeat it using a disbelieving tone, which was an indication that the students should go for the other answer. The following extract is taken from a Year 10, Set 3 lesson on trigonometry taught by Hilary Neville.

Hilary says, "Yvonne, part b?" Yvonne says, "Miss I can't do it." Hilary responds saying, "Well what is DC to the angle?, opposite or adjacent?" Someone calls out "opposite." Hilary continues, "and we've just found B, which was what?" Someone offers "adjacent." Hilary continues, "and opposite and adjacent give us what?" (pointing to some trig ratios on the board). Someone offers "tan." Hilary asks, "So tan what?" There is silence, so Hilary says, "tan 1.5" then "tan 1.5 gives us what?" Someone puts this into their calculator and gives the answer "14." Hilary says "correct" and moves onto the next question (Year 10, Set 3, Hilary Neville).

The teachers at Amber Hill cared deeply about their students and they clearly wanted them to succeed. When students asked for help with their questions, the teachers did not ask them what they thought they should do. Instead they gave students a series of instructions taking them through the questions:

M: He says you do this to get that, you do this to get that and you go "oh, right then."

H: Yeah, he gives you the answer, you write the answer down and that's it. (Helen & Maria, AH, Year 10, Set 1)

To help the students, teachers constructed paths that consisted of short, structured questions. These paths formed the basis of much of the mathematics guidance at Amber Hill. The teachers broke problems down for students and gave them lots of help because they believed this would give them mathematical confidence and, ultimately, help them learn mathematics. But the students and teachers seemed trapped within a vicious circle: The teachers thought that students would not or could not think. As a result, the students did not learn to think, and so the teachers' views were confirmed. In this way, the students' "learned helplessness" (Diener &

Dweck, 1978, p. 451) was continually reinforced. In interviews, the Amber Hill teachers seemed unaware of these tendencies. They reported that it was important for the students to find their own ways of solving problems, but in the day-to-day realities of the classroom, they rarely allowed this to happen. The teachers seemed driven by the need to seek and hear correct answers. This seemed to be due to two important factors. First, they were concerned to get through as much work as possible and therefore did not have time to spend letting students grapple with problems:

> Edward: You're very stringent to a time limit, you haven't got the time, like, you couldn't spend, there's certain things you have to sit down and tell them. I could spend a week letting them work through on their own, or, I know this group, I could explain it to them in one lesson and they'd understand it, which one do you do?

Second, the teachers seemed to take this approach because they believed that students would experience failure if they did not structure work for them. The teachers were influenced in this regard, by the fact that most of the students were from working-class homes, a point to which I return later.

Standard Mathematical Methods

At Amber Hill, the mathematics teachers began lessons with a presentation from the board of the mathematical methods, which students were intended to use in the exercises that followed. Teachers introduced students to the different procedures in a clear and structured way. However, they did not discuss their choice of mathematical methods, nor did they discuss with students when or why they worked. Students were not encouraged to discuss alternative approaches to problems or try their own methods. Indeed many students reported in interviews that they were actively discouraged from using their own methods:

> JB: Do you get the impression in maths lessons that there is one method that you're meant to follow or do you get the impression that there are lots of methods that you could use?
>
> P: No, there's just one method, her method.
>
> D: In school you have to use the method you are told to do. (Danielle and Paula, AH, Year 9, Set 2)
>
> C: Normally there's a set way of doing it and you have to do it that way. You can't work out your own way so that you can remember it. (Carly, AH, Year 10, Set 1)

The teachers at Amber Hill were keen to tell students about methods and strategies that were effective, but they neither placed these within a wider picture nor acknowledged the value of different or adapted approaches. The students' belief in the importance of learning the teachers' methods meant that they endeavored to remember these even when their own methods held more meaning for them.

JB: Does the method that's given to you make sense to you?
J: Not as much as my own.
JB: Your own method makes more sense?
J: Yes.
JB: Why do you think that is?
J: I dunno, you . . . I dunno. (Jackie, AH, Year 9, Set 1)

The teachers' concern to impart standard procedures meant that when students asked for help with questions, teachers would reiterate the procedure they should be using, rather than discuss the procedure or ask students to think about it. This was because the Amber Hill teachers regarded their major role in the classroom to be teaching the students mathematical methods, rules, and procedures. They did not regard the teaching of procedures as *different* from the development of sense making or understanding, and they did not perceive any need to teach anything other than their own standard or canonical methods. This was not a function of the broad teaching approach they chose—that of demonstrating procedures and asking students to use them to solve exercises. Teachers could have chosen to introduce procedures to students at the board, but also have encouraged students to understand them by asking probing questions. They could have asked students to think of other approaches that worked or pushed them to locate the methods within a broader mathematical domain. Instead the teachers chose to reiterate procedures and provide structure, rather than push for a wider meaning. The reasons for such choices are explored shortly.

The teachers' belief in the need to teach clear, standard methods also meant that they did not spend time linking different areas of mathematics or giving students an overall picture of the way different methods fit into the mathematical domain.

JB: Does he talk to you about the way things are connected, do you talk about maths generally?
L: Not really, you just do bits, you just do one topic, then another. (Lindsey, AH, Year 9, Set 4)

A: One day or one week we're doing one thing and the next week we go onto a different topic. (Anna, AH, Year 10, Set 2)

Lessons generally involved a sequential presentation of disconnected topic areas, which would be presented to students, one after the other, without any mention of any possible connections between them. The following notes were taken during a Year 9, Set 4 lesson with Edward Losely.

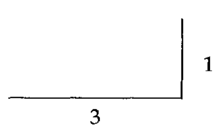
> The students all watch the board as Edward writes "b = 30 because they are *alternate* angles." Carlos shouts out, "what does that mean sir?" Edward says, "it means alternate." He then announces, "Right we're going to move onto something else." Daisy sighs and says, "I need a break sir." Edward ignores this and says, "Textbooks out please, page 91" and writes *Metric Units* on the board. (Year 9, Set 4, Edward Losely)

Here Edward demonstrates his concern to move onto a new topic, which prevented him from explaining a term to Carlos. This sudden change in direction was not unusual. Amber Hill mathematics lessons derived their form from the artificial structure of a textbook, which resulted in a somewhat disconnected presentation of mathematics. Hiebert and Carpenter (1992) suggest that connection making in mathematics is central to the development of mathematical understanding and question whether students should be told about connections or given the opportunity to discover them for themselves. Such issues did not form a part of the mathematics department's concerns at Amber Hill, and students were not encouraged to do either of these things.

Rules to Remember

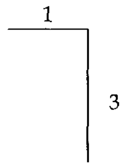
In many instances, I noticed that teachers actively discouraged students from thinking about mathematical relationships by telling them rules that they should remember. For example, the following extract is taken from my notes of a Year 8, Set 3 class with Tim Langdon.

> Halfway through the lesson, Tim raises his voice above the low level of noise and tells everybody to listen; he then draws a figure on the board:



> "If this is a line 3 long and 1 up what happens after a 90° rotation?" he asks. Some students shout out some answers: "It goes round," "left," "right" are shouted out by three boys. Another boy makes a joke of this: "It's up, down,

> left, right, north, south, east, west." Tim tries again, hears another boy shout out "it goes up," and responds, "yes, it does this doesn't it?" He then draws:



> The students all look at the new drawing but do not respond. "See what's happened?" Tim asks. "They've swapped around, the 3 goes up and the 1 goes across, so remember, when you do a 90° rotation you just have to remember to swap them round." The class listens to this instruction and then goes back to work (Year 8, Set 1, Tim Langdon).

In this extract, Tim told the students to stop thinking about what happened during a rotation and remember a rule. The object of the SMP exercise was to get students thinking about the movements during rotations and to try them out for themselves. Tim discouraged the students from thinking about the movements; he gave them something to learn instead. The teachers gave the students these rules because they believed they would help them.

The simplification of mathematical principles or methods to a set of rules was common at Amber Hill. When Hilary taught trigonometry to her students, she told them to learn the mnemonic SOHCAHTOA, as many teachers do, but she also gave the students other strict procedures that they should remember. The following extract is taken from one of Hilary's lessons on trigonometry, when the students are telling Hilary they do not understand:

> A lot of the students are chatting now; many of them are getting their questions wrong and seem to be very confused. So Hilary says, "Look, Lindsey, if you have a problem with a right angled triangle what is the first thing you have to find?" This is a strange question and I am not sure of the answer to it, nor are the students. Sue tries: "The angle?" Hilary says, "so what do you do?" Sue offers, "sin?" Hilary obviously feels this isn't leading in the right direction and so starts again with, "What is the first thing you've got to do with a right angled triangle?" Someone suggests, "know the two sides?", which seems to satisfy Hilary. She says, "yes, you've got to name them, you've got to know what the sides are" (Year 10, Set 3, Hilary Neville).

In this part of the lesson, Hilary tries to deal with the students' confusion by reducing the mathematical situation to a procedure the students should learn. The first part of this procedure was: "When you see a right angled triangle you label all the sides." Students were intended to learn this so they would label the sides of any right-angled triangles they saw, rather than interpret the particular situation they were placed in and decide what information they needed.

Hilary and the other teachers gave the students these handy hints or rules to make mathematics questions easier and more straightforward for students. The teachers gave the students these rules because the teachers understood the mathematics they were talking about. From that base of understanding, the rules appeared to be helpful to them. But the students did not understand the rules they were learning or the way that these rules related to the different situations they encountered. They did not locate the rules within a broad mathematical framework, and they did not develop a clear sense of what they meant.

> All of maths is just sums, rules and equations and none of it makes sense. (Bridget, AH, Year 9, Set 3)

I would suggest that this sort of mismatch between what the teachers and students gain from different rules underlies much of the confusion that students experience in secondary mathematics classrooms. Mathematics teachers understand what they are discussing, and they often give students structured procedures to learn to simplify and exemplify mathematical concepts. But the students do not regard these procedures as particular ways of thinking about the problems or as examples of the methods in action. They view the procedures as abstract rules to be remembered (Boaler, 2000a). Rules may be easy to learn, but difficult to use if they have not been placed within a wider sphere of understanding. Holt (1967) asserts that most teachers are driven by a desire to compartmentalize and provide models and structures that make sense for teachers but often do not for students. Mason (1989) talks about a similar problem: "To the teacher they are examples of some good idea, technique, principle or theorem. To students they simply are. They are not examples until they reach examplehood" (Mason, 1989, p. 2).

The Pace of Lessons

In Years 8 to 10, the students were taught from the front of the class at a fixed pace, as is normal for setted classes in England (Dahllöf, 1971). In the majority of cases, the pace of lessons was quite fast, and all of the teachers demonstrated a concern to keep the students working through exercises quickly. This derived from a desire, on the part of the teachers, to complete as many SMP textbooks as possible, cover all of the necessary content, and satisfy the demands of the national curriculum. Teachers also spent a lot of time in between lessons worrying about the speed at which their class was completing books; they would often discuss this with colleagues, asking where their class "was up to." Even when the teachers were explaining methods from the front of the class, they would often refer to the speed at which they were working, saying that they wanted to "just quickly" demonstrate something. This was particularly prevalent in the highest ability groups. The following notes are taken from a Year 9, Set 1 lesson taught by Tim Langdon.

> Tim arrives and immediately rubs some work off the board and says, "OK, quadratic functions, we began, last lesson, very quickly, with $x^2 - 3x - 4$." While he writes this on the board, the class watches and listens in silence. "And we said yesterday, how did we write this? Sara, you were the star yesterday." Sara looks at him blankly. Tim says, "anyone?" They all look at him blankly. He moves on quickly saying, "no-one knows? well it was $(x + 1)(x - 4)$." He writes this and continues, "From the book yesterday, we were practicing C1 yeah?, and C3?" Sara says, "Sir we got stuck on e." Tim picks this up saying, "Stuck on e?, well what number goes with x?" (the expression in question e is $x(x - 5)$). Eventually someone says "nothing." Tim says, "yes so the curve is $(x + 0)(x - 5)$, so nothing is nought, OK, C5, C6, so . . . C5a, what numbers will we get? Karina? (silence), Tafaz? what did you get?" Tafaz says, "I didn't get nothing cause I didn't do it." Tim continues, "well, what is the number?" Tafaz says, "I dunno I can't do this chapter." Tim moves on. "Sara, what is the number?" Sara says, "4 and 3." Tim comes back with, "so what do they give you?" Sara says, "12" and Tim starts to draw a curve on the board. All of the students are watching and listening in silence. So far all of this lesson has been delivered at breakneck speed, and I am not sure whether many of the students are understanding the concepts Tim is discussing. They can answer his small questions each time, such as "what do 4 and 3 make?" but I do not know how much more than this they are understanding (Year 9, Set 1, Tim Langdon).

Part of the teachers' desire to move quickly through work meant that when they questioned students from the board, they did not waste time on students who could not provide correct answers. On numerous occasions, I witnessed the different teachers speeding through demonstrations on the board and asking students questions, moving quickly around the class until they heard the right answers. The higher the set that the students were in, the more likely the students would be to get this fast and intense mathematical experience. These tendencies all created an impression that speed was very important in mathematics. Schoenfeld (1988) reports that this does not only put pressure on students, but it shapes their perceptions of what mathematical thinking involves. He found that students believed that mathematics questions should be answered within about 2 minutes—if they took any longer than this, they must have been doing the questions wrong; the implication being that mathematics involves working quickly, not thinking about questions deeply (Schoenfeld, 1988).

In Year 8, all the groups (n = 163) completed a questionnaire. This did not ask about the pace of lessons, but an open section asked the students to describe their mathematics lessons. This prompted 26 students (16%) to say that lessons were "too fast." Typical comments from students were:

> We are pushed hard to get work done and we work constantly at a fast pace.
> The teacher rushes through methods faster than most pupils can cope.

The speed at which teachers took students through their work had an impact both on the way students viewed mathematics as well as their learning of mathematics. Both of these responses are considered shortly.

The Teachers' Motivations

The Amber Hill teachers were strongly motivated, with good intentions, to reduce the complexity of mathematical thought. This influenced their whole approach to mathematics teaching, causing them to close problems down, emphasize set methods and procedures, keep different topic areas distinct from each other, and give students rules to remember. These approaches fitted in with the teachers' general philosophies about mathematics teaching, but there was evidence that the teachers had made their teaching *more* procedural and *more* rule-bound because of the social class composition of the school. Amber Hill was a largely working-class school, and the teachers had low expectations for their students; in particular, they felt that the students had a reluctance to think for themselves or use their initiative.

> Tim: Students are generally good unless a question is slightly different to what they are used to, or if they are asked to do something after a time lapse, if a question is written in words or if they are expected to answer in words. If you look at the question and tell them that it's basically asking them to multiply 86 by 32 or something they can do it but otherwise they just look at the question and go blank.

The different mathematics teachers seemed to share the belief that the students were incapable of complex mathematical thinking, but they did not relate this observation to the approach they offered at the school or to prior mathematical experiences, but to the students. In particular, features related to their background:

Tim: I think there's a paucity of language here that the kids are using, that I think causes the problem, having taught in Hertfordshire *(a more wealthy area)* with much more breadth, with, if you like a professional background, there was higher performance there. (words in italics added)

Leisel: I think the reading is a big problem with our children, they don't want to think about what they've read, then they'll say I can't do it, I don't understand it and I think that's where it all breaks down as well. They have learned maths but they can't be bothered to think about it. It's got to do with ability and motivation as well, because in this school we have a lot of pupils who have very little motivation, you know? They're not encouraged at home.

Hilary: I think textbooks are better for the pupils we've got, I think they get more advantage out of it. I think there's more motivation than—they don't need as high a motivation for a textbook than they do for individualized learning. And I think for the type of pupil we've got and parent, it's better that way.

The teachers' belief in the inadequacies of the students at the school made them think that a low-level structured approach would be most appropriate for them, and this approach did not conflict with the teachers' views about good mathematics teaching. Anyon (1981) cites a number of studies (Keddie, 1971; Leacock, 1969; Rosenbaum, 1976; Sharp & Green, 1975) finding that schools in poor and working-class areas "discouraged personal assertiveness and intellectual inquisitiveness in students and assigned work that most often involved substantial amounts of rote activity" (p. 203). One of Anyon's studies found that mathematics teaching in working-class schools was procedural, rule-bound, and involved the learning of set methods by rote. In more middle-class, professional, and elite schools, the mathematics teaching involved choice, analytic reasoning, discussion of different methods, and emphasis on mathematical processes (Anyon, 1980). Amber Hill school conformed to this pattern, and the teachers' approaches in the mathematics classroom seemed to derive partly from their views about the limitations provided by the students' home backgrounds. The teachers were not unusual in believing that students from low-income homes require more structure. Lubienski (2000) recently offered a similar proposal having taught a class of students using a reform curriculum. She found that some of the working-class students had difficulties with the open work and concluded that open approaches may be inequitable. These are extremely important issues to discuss. It is essential that we collect more evidence on the relationship between teaching approaches and equity, but I have argued that Lubienski's response is limited by the fact that it draws direct links between the curriculum and achievement, with no analysis of the *teaching* that makes the difference between equitable and inequitable approaches (Boaler, 2002a).

THE STUDENTS' REFLECTIONS

Enjoyment

In all of my interviews with the 40 Amber Hill students, I received negative reports about mathematics lessons even when students were chosen for interview because they had been reasonably positive about mathematics in questionnaire responses. This was not due to any prompting on my part. I generally started interviews with, "Can you describe a typical maths lesson to me?" This was usually enough encouragement for the students to describe all of their negative feelings about mathematics. The extracts that follow have not been chosen to give a negative impression of the teaching the students experienced, nor have they been pulled out of transcripts with more positive reflections that I have ignored. When I interviewed the Amber Hill students, I uncovered a large degree of disaffection. I also discovered that the students were able to talk extremely coherently and analytically about their learning experiences and the conditions that would make their learning of mathematics more productive. The reasons that the students gave for disliking mathematics, in interviews, were also consistent with those given in questionnaires and classroom conversations. These related to the lack of variety in the school's approach, the lack of opportunities to think about mathematics, and working as a class at a fixed pace.

Variety in Lessons. The students at Amber Hill gave various indications that they were bored by their mathematical experiences. In their Year 8 questionnaire, students were asked, "What do you dislike about the way you do maths at school?" Forty-nine students (31%) criticized the lack of variety in the school's approach, and 77 students (48%) reported that they would like more practical or activity-based work. Typical comments were:

"Maths would be more interesting if there were more projects to do."

"I don't think we should work on boring textbooks all the time."

"The way we always look at the same old textbooks (boring) and never change systems."

The students were dissatisfied not only because they worked through textbooks for the vast majority of the time, but because they thought the questions within the books were very similar:

S: The books are a bit boring, the chapters aren't really that good and they repeat the same questions over and over again, like when they explain something they do the question and then you have to do about twenty of them at the same time.

G: Yeah and you only needed to do one, to know what's going on. (Steven & George, AH, Year 9, Set 3)

The students did not blame their boredom on the intrinsic nature of mathematics. They were aware that they could gain enjoyment from learning mathematics because they liked their coursework lessons and most of them had enjoyed their elementary school mathematics. The students merely felt that it was inappropriate and unnecessary to work through textbooks all of the time, and they wanted more variety in their mathematics teaching:

JB: If you could change maths lessons what would you do?

R: I'd have maybe one lesson a week on the booklets, one on activities, one where you get a problem and you have to solve it—just a variety. (Richard, AH, Year 10, Set 2)

The students were far from unreasonable in their requests. In their Year 10 questionnaire, the students were asked what they liked about mathematics lessons. The most popular response—given by 50 students (31%)—was, "I like maths when we do activities,", whereas only 4 students (3%) said that they liked their textbook work. When asked what they disliked about mathematics lessons, the four most common responses were: working from books (22%), not understanding (20%), work being all the same (19%), and work being boring (17%).

Open-Ended Work. In their textbook lessons, the students did not think they were able to develop their ideas or use their initiative. They became aware of the value of these features of their learning when they were given open-ended coursework projects to work on for 3 weeks of Year 9. These were projects that involved mathematical decision making in real-world situations, such as "planning a day trip." The students believed that the openness they experienced within coursework made mathematics more enjoyable, but also helped them to learn:

D: I feel restricted when we're doing the books.

R: Coursework is better than the book-work you know, because with coursework you could go out and you can just—you learn more by doing something on your own, you know, if you're doing something on your own, you learn, well I found I learned more by doing something on my own than I had done with the teacher. (Richard & David, year 10, set 2)

S: It's a better way to learn.

JB: Why is that?

S: 'Cause I can figure it out for myself, the books just, it's too much leading you through it. (Sacha, AH, Year 10, Set 4)

The students described their coursework in terms of an increased cognitive demand. They did not regard coursework as an easy option, and for many it meant a lot of effort and hard work, but they valued this experience because it allowed them to *think* and feel ownership of their mathematics in ways that textbooks did not:

L: It was a project, so it was going from one little thing and getting this big result at the end—working through on your own, going through different stages I was really proud of it actually, it was good.

S: We was dead chuffed weren't we? *(very pleased)*

L: You feel more proud of the projects when you done them yourself. If it's just working through the book, you can't feel proud—well, you can get them right and nobody cares—like you've seen it, it doesn't really matter, but if it's like a big project and you can see like what mark you've got at the end and if you've worked hard and if you get a good mark you feel really good about it. (Sara & Lola, AH, Year 11, Set 3)

The students felt a sense of ownership for their coursework projects, which they related to the amount of effort they had put into their work and the requirement to think about what they were doing. In the textbooks, the students were "led through it"; they were not allowed to "work things out," and they felt "restricted." The students were clear that the openness of coursework enhanced their learning. Students were asked in their Year 8 questionnaire (before they had encountered coursework) to describe the "most interesting piece of mathematics" they had ever done in a lesson, and almost half of all students (49%) cited the same mathematical experience – using Logo (a programming language) on the computer. When they completed a similar questionnaire in Year 9 and were asked to describe their favorite lesson, 62% of students chose their open-ended tasks and 9% chose computer activities. A further 17% either left the space blank, said they could not name a good lesson, or described something unrelated to mathematics, such as the teacher being absent. Of the students who actually described a mathematical experience, 81% chose their open-ended task as their best ever mathematics lesson.

Working at Their Own Pace. When the students began Year 8 at Amber Hill, they had just experienced 2 years of working through individualized booklets at their own pace. Students at both schools used this approach in Years 6 and 7. The individualized booklet approach, which is part of the SMP curriculum, was partly designed to enable mixed-ability teaching: All the students work through a sequence of small booklets at their own pace, and different students can work on different booklets at any one time. The teacher does not instruct from the front, but wanders around and helps students as they work on their questions. For many students, the change from this system to a system whereby the whole class worked through pages of a textbook at the same speed was quite a shock. In interviews conducted in Years 9 and 10 at Amber Hill, working at the pace of the class was a major complaint for almost all of the students and one variously related to disaffection, boredom, anxiety, and underachievement. Many of the students were unhappy because they felt that the pace of lessons was too fast. This often caused them to become anxious about work and to fall behind, which then caused them to become more anxious. This response was particularly prevalent among the top-set girls. However, the anxiety caused by fixed-pace lessons did not only prevail among the top-set students or girls. In the following, the students all relate the fixed pace of lessons to a loss of understanding:

A: I preferred the booklets.
S: Yeah 'cause you just get on with it don't you?
A: Yeah, work at your own pace. You don't have to keep up with the others.
JB: Do you feel that now?
A: In a way because if you don't do all the work, then you get left behind and you don't understand it. (Suzy & Anna, AH, Year 10, Set 2)

L: You don't learn it, you're just rushing and trying to make sure you get it done just so you don't get in trouble and you can catch up with everyone else. (Lindsey, AH, Year 10, Set 4)

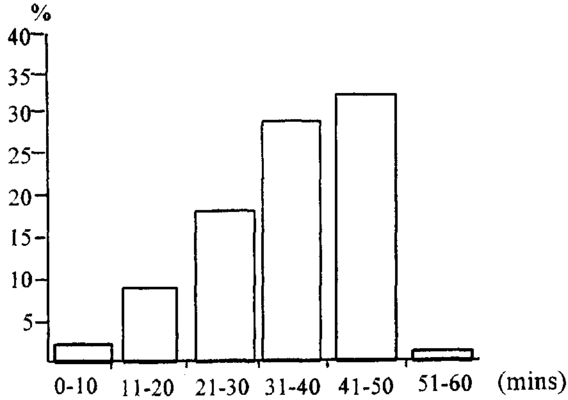
The majority of students related their reservations about class teaching to what they perceived as a resultant loss of understanding. However, while some students, predominantly girls, complained about the fast pace of lessons, other students in the same groups said that their learning was diminished because lessons were too slow. These were usually boys:

> M: It's silly now, it's just, most of the people slow the class down, gets it more boring.
>
> C: You don't learn as much.
>
> M: Like people laze around, when they've completed the work ... say we've completed the work and we can go further up the book, we have to do that piece of work and then stop, and wait for the others to catch up and then people laze around. (Chris and Marco, AH, Year 10, Set 4)

Some students complained about the pace of lessons being too fast, whereas other students in the same classes complained about lessons being too slow. This discrepancy reveals an important limitation of a teaching approach that requires all students to complete the same work at the same time. For the teacher, it shows how difficult it is teaching a group at the same pace even when they are meant to be of "homogeneous" ability. Amber Hill divided the students into eight sets, which should produce relatively little variation among students in the same set, yet the students reported that the variations among them caused problems. The complaints of the different students at Amber Hill may also reflect that the ability of a student does not necessarily indicate the pace at which they feel comfortable working, although this is an assumption on which class teaching to setted groups is predicated in England. Further consideration of the implications of being in setted groups for the students at Amber Hill is given in chapter 8.

Engagement

In the vast majority of lessons that I observed, students showed a marked degree of uninterest, uninvolvement, and boredom with their work. Passivity was commonplace, demonstrated by rows of students quietly copying down methods without any apparent desire to challenge, question, or think about their work. This was the way the students responded to what they perceived as the boredom of lessons. In Corrigan's (1979) study of working-class boys and their responses to schooling, he found that *mucking about* was a major activity in classrooms and not paying attention was endemic. Many of the Amber Hill students did not pay attention during substantial parts of lessons, but they normally



n = 147, mode = 45, mean = 37.7

FIG. 4.1. Students' perceptions of time spent working at Amber Hill.

confined their "mucking about" to quiet, nonmathematical conversations with friends. When I recorded the number of students working in lessons, over 90% of students appeared to be on task at three different times, but when I asked all the students to anonymously write down how many minutes they worked in lessons, the average of all of the times given by the 147 students in the case study year group was 38 minutes. The difference between my records of time on task and the students' perceptions of the time they spent working in lessons was partly due to the fact that students made sure that they looked as if they were working when they were not. It may also have been due to the fact that the students often worked through exercises they were given to do without any thought or involvement:

A: So we do equations and formulas, like roughly the same thing you do and you don't even like think about what you're doing, you just do it 'cause it has to be done. (Alan, AH, Year 10, Set 3)

K: As soon as you walk out the class . . . well actually as soon as the classroom starts, you don't really know anything, 'cause you've switched off. You walk in and you think, oh another boring lesson and you're off. As soon as you've walked out, you've forgotten about that lesson. (Keith, AH, Year 10, Set 7)

The students often worked because they thought they had to, not because they enjoyed their work or because they were engaged with the mathematics. This meant that they were often working without thinking:

C: I think people start to think . . . oh, I hate it, but we've got to do it, we haven't got much choice . . . I think that's the thing that keeps people going on most of all. Like if you asked people is maths your favorite subject, hardly anyone would say it is, but they know they have to do it, cause it keeps getting drilled into them that you need maths, it's a good qualification. People think oh well I've got to do it so I might as well do it. (Carly, AH, Year 10, Set 1)

The students' attitude of, "I might as well do it" was not conducive to their learning, and the students were aware that they could work in mathematics lessons without gaining much from it:

M: Yeah it depends if I'm in the mood, but I think, if it's like a lesson when I decide to work hard and I do work hard then I find that I succeed more, and I understand it more really, rather than if I just do it because I've got to do it. (Maria, AH, Year 10, Set 1)

D: Coursework was better because you could spend time on that and get involved with it, and you worked because you wanted to. (David, AH, Year 10, Set 2)

The difference the students highlighted between working when they wanted to work and working because they had to is an important one. This is partly because this distinction may underlie the difference between learning and working procedurally. Almost all of the students talked about the time they spent in mathematics lessons "switched off" and working without thought. In a sense, the Amber Hill students were exercising their own style of control over their work by not thinking—the only control open to them. This difference between learning and working without thought is also important because it raises questions about the validity and usefulness of time-on-task assessments (Peterson & Swing, 1982). The students at Amber Hill would have looked to anybody as if they were hard at work, but their assessments of the time they spent working and their comments in interviews show they spent much of their lessons with their minds elsewhere. The distinction the students drew between engaged and nonengaged work is also important because it suggests that the preoccupation teachers often have with keeping students quiet and orderly (Doyle & Carter, 1984) may not be justified. The Amber Hill students said they were engaged when they believed an activity to be worthwhile; at other times, they would work, but got little out of it. This suggests that the nature of tasks that students are given to do is far more important than keeping them quiet and on task, and that high-risk tasks (Doyle & Carter, 1984) that increase classroom disorder may ultimately be worthwhile.

Students' Views About the Nature of Mathematics

Students at Amber Hill appeared to have developed clear and consistent views about the nature of mathematics, the significance of which emerges in later chapters. One important view that was common among students was that mathematics was a rule-bound subject.

Rule Following. Many of the Amber Hill students held a view that mathematics was all about memorizing a vast number of rules, formulas, and equations. They did not believe that mathematics was a rich or varied subject, nor did they regard it as a "doing discipline" (Treffers, 1987, p. 60). They thought that their job in the mathematics classroom was to learn rules:

> A: At the end of each chapter if they had a list of rules it would be so much easier, like now, I'm revising *(reviewing for a test)*, I'm trying to go through the book and I'm looking for the rules, if they had the rules at the end it would be better . . . I bought a revision book from the school and they've got a few rules in it but again they sort of, you know, you've got to try and find the rules, they're not all set out for you. (Alan, AH, Year 10, Set 3)

The students' belief in the need to remember rules had an important influence on their mathematical behavior. As a result of approximately 100 lesson observations at Amber Hill, I defined two main behaviors that seemed to influence the students' mathematical decision making. I termed one of these *rule following* because when the students approached new situations they did not try to interpret what to do; rather they tried to remember a rule they had learned. Part of the reason students did this was that they thought it was inappropriate to try and interpret the particular situation given to them because there was only one specified way to solve each question and this involved remembering a rule:

> S: In maths, there's a certain formula to get to, say from a to b, and there's no other way to get to it, or maybe there is, but you've got to remember the formula, you've got to remember it. (Simon, AH, Year 10, Set 7)

> L: In maths you have to remember, in other subjects you can think about it. (Lorna, AH, Year 10, Set 1)

The students not only believed that there were a lot of rules that could be learned in mathematics; they also believed that they *had* to remember these rules to solve questions. Some of the students were so convinced of this they did not see any place for thought within mathematics lessons. The predominance of the students' belief in the importance of remembering rules was further demonstrated by the Year 9 questionnaire, which I devised in response to my fieldwork. In one item of this questionnaire, students were asked which they believed to be more important when approaching a problem: remembering similar work done before or thinking hard about the work at hand. Almost two thirds of the students (64%) said that remembering similar work done before was more important than thinking hard. This view appeared to be consistent with the strategies they employed in class and was, in many ways, indicative of their whole approach to mathematics. The Cognition and Technology Group at Vanderbilt (1990) note that when novices are introduced to concepts and theories, they often regard them as new "facts or mechanical procedures to be memorized" (p. 3). The Amber Hill students rarely seemed to progress beyond this belief.

There were many negative consequences of the students' belief in the rule-bound nature of mathematics. One of these was that their desire to remember different rules meant that they did not try to interpret and understand what they were doing. Thus, they would learn rules and use them in situations to which they could easily be applied, but when the situations changed they became confused. A second negative consequence was that when students encountered questions that did not require an obvious and simplistic use of a rule or formula, many did not know what to do. In these situations, they would give up on questions or ask the teacher for help. A third problem was provided for the students, who thought that mathematics *should* be about understanding and sense-making (Lampert, 1986). These students experienced a conflict at Amber Hill because they wanted to gain meaning and understanding, but felt this was incompatible with a procedural approach:

JB: Is math more about understanding work or remembering it?

J: More understanding, if you understand it you're bound to remember it.

L: Yeah, but the way sir teaches, it's like he just wants us to remember it, when you don't really understand things.

JB: Do you find that it is presented to you as things you have got to remember, or is it presented to you as things you have got to work through and understand?

L: Got to be remembered.

J: Yeah remember it—that's why we take it down in the back of our books see, he wants us to remember it. (Louise and Jackie, AH, Year 9, Set 1)

The students who wanted to understand their mathematics in depth were mainly girls, which is discussed further in chapter 9. In many ways, these girls were more disadvantaged than the boys, many of whom were happy to just learn the rules and *play the game.*

The students' belief in the need to follow rules caused problems for them because it had an enormous impact on their behavior. The students were confined by this belief, and in new situations they did not try to think about what to do. Instead they tried to remember a rule or method they had used in a situation they thought was similar. However, because in mathematics lessons they were not encouraged to discuss different rules and methods or think about why they may be useful in some situations and not others, the students did not know when situations were *mathematically* similar. This was part of the reason that they developed a form of behavior that I have described as cue-based (Schoenfeld, 1985).

Cue-Based Behavior. Frequently during lesson observations, I witnessed students basing their mathematical thinking on what they thought was expected of them, rather than on the mathematics within a question. The students would use a range of nonmathematical cues as indicators of the teacher's or textbook's intentions. These sometimes related to the words of the teacher, but students would also use such cues as the expected difficulty of the question (what they thought should be demanded of them at a certain stage), context of the question, or teacher's intonation when talking to them. The following extract is taken from my field notes of a Year 8, Set 1 lesson:

> After a few minutes, Nigel and Stephen start to complain because there is a question that "is a science question, not a maths question." They decide they cannot do it, and I go over to help them. According to the problem, 53% of births are male babies and 47% female babies, but there are more females in the population. Students are asked to explain this. I ask Stephen if he has any idea, and he says, "because men die quicker." I say that this is right and leave them. Soon most of the students are putting their hands up and asking for help on the same question. Carol, a high-attaining girl, has already completed all of the exercise, but has left this question out and says that she cannot do it.
>
> Later in the lesson, Helen has her hand up and I go over. The question says that "58.9 tonnes of iron ore has 6.7 tonnes of iron in it. What percentage of the ore is iron?" While I am reading this, Helen says "I'm just a bit thick really." I ask Helen what she thinks she should do in the question, and she immediately tells me correctly. When I tell her that she is right, she says, "But this is easier than the other questions we have been doing: in the others we have had to add things on and stuff first." A few minutes later, two more girls ask me for help on the same question: Both of these girls have already completed more difficult questions (Year 8, Set 1, Edward Losely).

These two examples demonstrate different forms of cue-based behavior. Nigel and Stephen and all of the other students who stopped working when they reached the question on babies stopped because the question required some nonmathematical thought. They could do the question, but they thought their ideas must be wrong because they did not expect a question with science in it in a mathematics lesson. The girls gave up on the question on iron ore because the mathematical demand was different from what they had expected. The previous exercise presented a series of abstract calculations in which the students were asked to work out percentages that required them to "add things on and stuff first." In the next exercise, the questions were mathematically simpler, but they were contextualized. The writers of the textbook obviously regarded these as more difficult, but the girls were thrown by this because they expected something more mathematically demanding. This expectation caused them to give up on the question. It is this sort of behavior that I have termed *cue-based* because the students were using irrelevant aspects of the tasks, rather than mathematical sense-making or understanding, to cue them into the right method or procedure to use. The students developed a whole range of cue-based practices in their mathematics lessons. For example:

- When working through exercises, students expected to use the method they had just been taught on the board. If a question required the use of a different method, they would often get the answer wrong or become confused and ask for help. This occurred even when students knew how to use the required method.
- When working through exercises, students expected later exercises to require a slightly more difficult method than the previous exercise.
- If a question required some real-world knowledge or nonmathematical knowledge, students would stop and ask for help. They would be able to answer the question if prompted; they would probably be able to answer the question if they were in a science classroom or if they were at home, but their expectation of the knowledge they should use in a mathematics classroom stopped them from answering such questions.
- The students always expected to use all of the numbers given to them in a question or all of the lines present on a diagram. If students did not use them all, they thought they were doing something wrong and changed their methods to ones that could include all of the numbers or lines.

These different examples demonstrate the codes of the mathematics classroom that students lived by—the implicit norms of their classroom that influenced their use of mathematics. If students encountered textbook situations that departed from their expectations, they became confused—not because of the extent of their mathematical knowledge, but because of the regularities of the mathematics classroom to which they had become attuned. Schoenfeld (1985) asserts that this sort of cue-based behavior is formed in response to conventional pedagogic practices in mathematics that demonstrate specific routines that should be learned. This sort of behavior, which was common among the Amber Hill students, meant that if a question seemed inappropriately easy or difficult, if it required some nonmathematical thought, or if it required an operation other than the one they had just learned about, many students would stop working.

The students used different cues—from the textbook and the teacher—to help them know what to do in different situations. In a sense, the students were forced to do this because they had not learned to interpret situations or think about them mathematically. Their cue-based strategies were also effective; they often allowed them to attain correct answers. It was only in unusual situations, where the questions did not fit into the usual textbook prototype, that the students became confused. Yet these classroom strategies were ultimately destructive because they worked against mathematical thinking. The methods discouraged sense-making and understanding, and they were completely ineffective in non-SMP and nonclassroom situations.

It also seems important to note that the cue-based reactions of the Amber Hill students emerged within the interactions of teachers, students, and curriculum (Cohen, Raudenbush, & Ball, 2000), and the students' important motivations could only be understood through a focus on the interactions that took place in the classrooms. The teachers did not tell the students to follow cues, nor did the textbook, although the teacher and textbook were important contributors to the responses that emerged. The students did not come into the classroom intending to follow cues, but the teacher, textbook, and students were all important participants. If I had chosen to focus on the teacher, students, or the curriculum, as much educational research has done, or if I had omitted to observe classrooms at all, as many of the proponents of the math wars have done, I would not have understood an important aspect of the students' behavior that emerged repeatedly in different mathematics assessments, as I explain shortly.

SUMMARY

In this presentation of Amber Hill's teaching, I have highlighted the closed, procedural, and fast-paced nature of the students' experiences. Such reflections were guided to a large extent by the students' own reflections on their learning. In the midst of this rather bleak portrayal of mathematics teaching, I hope that I have also given a sense of a mathematics department that cared deeply about its students. The Amber Hill teachers wanted to help their students learn, and the fact that they chose to enact their good intentions through a structured presentation of mathematics is not completely surprising. This is a common method of teaching that has been used by mathematics teachers for centuries. Such a teaching method may also be good for some students—it is not dissimilar to a model of teaching I experienced in school, through which I developed an understanding of mathematics. I doubt that it is dissimilar to the teaching these teachers experienced in school, which enabled their own success. But I, like a number of other students, essentially constructed my own understanding of the connections that comprise the mathematical domain. I thought deeply about the methods that were presented to me, and I solved open mathematical problems in my own time at home. I went beyond the procedures presented to me in class, and I am sure that some of the students at Amber Hill did the same.

The majority of school students do not do that, and if their teachers present a series of structured methods, their reasonable response is to memorize them, as the majority of Amber Hill students tried to do. The wide range of students I interviewed at Amber Hill spoke in consistent ways about their dislike of the structured approach they experienced and their enjoyment of the open work they occasionally met, but there were probably other students who held a different view. Despite the variability among students that may have been more present than I have represented in this chapter, it seems important to give careful consideration to the reflections of the Amber Hill students that I have reproduced here. The students gave clear messages about the features that impeded their learning, and these will become increasingly important to reflect on as the later chapters unfold.

In concluding this chapter, it is also important to note that the Amber Hill teachers gained the respect of their students, commanded their attention, and produced a high work rate from students, all of which are important features of teaching. That they did not look beyond the students' compliant reproduction of methods and push the students to make sense of the methods they were using is a criticism that I prefer to level at the opportunities (or lack of them) for teacher learning in England. Teachers in England receive few opportunities for professional development and for learning about the needs of students who are different from themselves. Those opportunities that do exist for professional development tend to operate on a generic level, without addressing the needs of underserved students who live in economic hardship. The reluctance of the Amber Hill teachers to push their students or create opportunities that would enable students to develop deeper understandings should also not be taken as a criticism of all teachers using more traditional methods of teaching. At the same time, there is considerable evidence to suggest that many mathematics teachers present the subject in the way that the Amber Hill teachers did. In the third international mathematics and science study (TIMSS), 45% of English 13-year-olds reported that memorizing textbooks was the key to success in mathematics (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997). After analyzing a cross-national sample of teaching videos, Stigler and Hiebert (1999) concluded that the type of approach characterized by the Amber Hill teachers in which students "spend most of their time acquiring isolated skills through repeated practice" (p. 11) is a deeply cultural phenomenon pervasive in the Western world. There is a lot of evidence that the Amber Hill approach is far from unique and that the students' reflections on the limits of the approach are extremely important to consider.

5

Phoenix Park Mathematics: Experiences and Reflections

In a similar style to the last chapter, the two main sections of this chapter describe some important features of Phoenix Park's approach and the students' responses to them. I again select the particular aspects of the teaching approach that seemed to influence students' views and understandings to the greatest extent, and that also give a representative portrayal of Phoenix Park's approach.

TEACHING AND LEARNING AT PHOENIX PARK

Open Learning

At Phoenix Park school, the curriculum was designed by the teachers. They did not use any books or work cards. Instead they brought together a collection of different open-ended projects that generally lasted for 2 to 3 weeks of mathematics lessons. Probably the most distinctive, influential, and unusual aspect of Phoenix Park's mathematics approach was the openness and freedom that this created for students.

> G: In books, it tells you everything, you read everything off the question, you read the question and you have to answer it. Here you just have to make up your own, he just tells you what you have to do and then you have to do it yourself. (Gary, PP, Year 9, JC)

The mathematics approach at Phoenix Park was open from the time projects were described to students to the time, 2 or 3 weeks later, when they gave them in. This openness manifested itself in a number of ways, including the ways in which the projects were described and defined, the ways in which teachers answered the students' questions and the ways in which teachers guided students. The students at Phoenix Park were not given specified paths through their activities; they were merely introduced to starting questions or themes and expected to develop these into extended pieces of work. When they asked the teachers questions, the teachers seemed to make deliberate efforts not to structure the work for students:

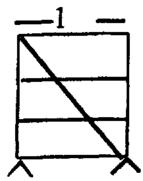
> JB: When the teachers help you here, do they talk to you generally about the topic or do they break it down and tell you bit by bit what to do?
> A: Very general, they hardly give you an answer.
> D: Usually it helps, 'cause then they don't really give you the answer, you still have to work it out for yourself. (Alex and Danny, PP, Year 10, JC)

Thus, the openness of the approach related not only to the way that mathematics was introduced, but also the way in which teachers interacted with students and supported them in their work:

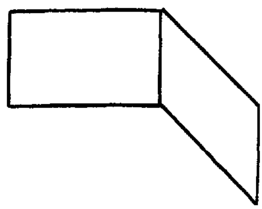
> A: Well, I think first of all you have to try and find your own methods, then if you really get stuck the teacher will come and give you suggestions for stuff and tell you how to like, progress further and then you can kind of think about it. (Andy, PP, Year 10, RT)

I have chosen the following extract because it gives a fairly representative example of the ways projects were introduced at Phoenix Park and illustrates some of the decisions teachers made as they introduced mathematical ideas and methods. In the extract, Jim is introducing a new activity called *36 pieces of fencing* to a Year 8 class.

> Twenty-five students come in and sit down. Sixteen boys and 9 girls gather around the board. A boy asks, "Sir are we gonna start a new project?" Jim says, "Yes, the title of the piece is *36 pieces of fencing* (he writes the title on the board), so you need a piece of paper — it only needs to be a scrap piece of paper at the moment, but make sure you've got something to write on." A few get up and collect paper from a stand in the room. Jim continues, "Can we have a bit of hush please? Right, you have a piece of fencing and from the side it looks like this:

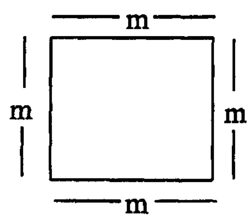


It has little legs on, like this and it is 1 meter wide. They have little hooks on and you can hook the gates together. You can put them at any angle, so those from the side would look like this":



He continues, "What we are interested in is what sort of shapes can you make with 36 pieces of fencing?" The students then start calling out shapes, a boy offers a square, a girl a hexagon. Someone asks, "do you have to use them all?" Jim says, "Yes, there are rules" and writes a heading: Rules on the board and then, under this, *use them all on every shape,* saying "it makes them more manageable if you have to use them all." Then, "any other shapes?" A girl says "rectangle." Jim asks, "just one?" A boy says, "a square is a rectangle." Jim says, "yes, we've already got a special type of rectangle." The students continue shouting out shapes. A boy says "rhombus," a girl says "parallelogram," and Jim is adding all of these to a list on the board. Another boy says, "pentagon," and Jim stops at this and says, "can you?" The boy says, "yeah." Jim asks, "how many sides?" A few offer "5." A girl says, "you've got 36 fences," and Jim says, "well you can have a pentagon, but what will it be like?" There is silence, so he asks, "will the sides be the same?" The students all shout "no." Jim asks, "so what will it be called?" A boy offers, "irregular." Jim writes *irregular pentagon* and then asks for more shapes. One boy suggests a quadrilateral and Jim says, "Yes, well, these are all quadrilaterals," and he points to some shape names. He puts parentheses around these on the board and writes *quadrilaterals* next to them. He then continues with "we've got a triangle but is there only one?" A girl says, "there's loads." Jim says, "yes there's loads so lets put an s on," and makes it triangles. Then Jim says, "so, we've got 4 sided, 5 sided. . . ." A boy offers "octagon" and a girl says, "yes, 8 sides." Jim asks, "yes, but what will happen?" Someone says, "there'll be some left over." Jim says, "yes, or irregular, not all the same length, so *pentagon*" and writes *(irregular) heptagon (irregular) octagon (irregular).* A boy offers "nonagon" and Jim tells him to say it louder so that everyone can hear. Jim writes it on the board with (9) next to it, then asks, "will it be regular or irregular?" A girl says regular and Jim asks why.

She says, " 'cause 9s go into 36." Jim asks, "what other regular ones can we have?" There is silence and he adds, "well the definition seems to be if the number of sides go into the number of fences." A boy says "12" and Jim writes *dodecagon (12 sided)*. Someone offers "18" and Jim says he doesn't know what that is called, but writes up *18-sided shape*. A boy says "3 sides," and someone else says, "that's a triangle." Jim asks, "OK how many regular triangles can you make?" Someone says "one" and Jim says, "yes, where I've written regular you can also have irregular ones." He then asks, "which are easier to draw?" Someone says "irregular" and Jim says, "OK shall I make it harder and say we only want regular ones?" Some say "no" to this and some say "yes." Jim says, "we can put another rule in if you want" and writes under the rules heading *only make regular shapes*, but then adds *(you can break it sometimes)*. Justin says, "I always break the rules sir," and Jim says, "really Justin." Then "now, tell me something about a square." A girl says, "they're all the same length." Jim says, "yes so I have to go round 4 lengths all the same and if I call this m," he draws:



and says, "I'll say 4 times m equals what?" A girl and boy say "36," and Jim asks, "so how do I work out what m is?" A few say "9," one girl says "36 divided by 4." Jim responds to these saying there are two ways of looking at it: "we can say $4 \times m = 36$ by thinking about our times table, or we can say 36 divided by $4 = m$, but you can only really use the first when it's a whole number." Then "so how big is it? what is the area?" A few say "81," and Jim says, "the area is 81 meters squared. Why meters squared? because it's an area, when you work out area it's meters squared." Then "I want you to look at all of those shapes and find ones that are possible to do, and I'm interested in the area of them, why might I be interested in area? what is it useful for? I may be making a garden or a pen." Jim suddenly turns to a boy near him who has been chatting incessantly and says, "Michael, it is irritating you talking all the time, OK?" Michael looks repentant and Jim continues. "So I'm interested in area. I'd like you to explore these shapes and find areas. Now, the first thing I'd like you to do is record what I've been talking about. My writing isn't sufficient; you need to put things in your own words, your version of the problem. Expand it, write what it means, pick out shapes, decide what order you need to do them in!" As the students go back to their seats and start working, Matt, who is new to the school, says, "sir, I don't understand these shapes, I don't think I've seen them before." Jim says, "well that could be one of your tasks, find out about the shapes, look them up in your maths dictionary, or you could look in an ordinary dictionary."

> Most of the class starts work; some are talking, some have started straight in with drawing squares, some check with Jim what they are meant to be doing, two boys sitting at the back are talking about something else and not working. Matt is looking up the shapes in a dictionary, most students seem to have started doing the task, without explaining in their own words what they have to do, as Jim suggested. Three boys sitting together at the back are slow to get started; they write a few words, talk for a while, write some more, and so on. Two boys pick up their table and move it round to avoid the sun. Jim is kneeling down by the side of somebody saying, "you draw it whatever size you want to draw it." Most of the students' introductions say, "we need to see how many shapes we can make out of 36 fences" or something similar. Four girls are sitting having an animated and excited conversation about all the different shapes: "is it a quadrilateral?"(laughs). Another says, "what's that?" Another says "a trapezium?" They seem very interested. As I pass Julie, she checks with me what a regular shape is before she writes out her definition in her introduction. Some students have now written about a paragraph. Three boys at the back have only written a heading and a sentence. Most of the rest of the class have moved onto examples. None of these students is using calculators, nor do they ask for them, although they are available. One of the boys is finding out which is bigger, a rectangular area or a triangular area. Jim comes over and says, "so which was bigger?" The boy tells him and he asks, "is that what you would think, does it look bigger?" They discuss this for a while. (Jim Cresswell, Year 8)

The previous extract is a fairly typical example of an introduction to a project on which the students worked for approximately 3 weeks. The only unusual aspect was that the students were given one project to work on, rather than a choice of projects. Jim's introduction incorporated a number of features that related to the openness of the approach. Jim introduced the problem of *36 fences* by getting the students to think about the different shapes that were possible. He did not spend much time at the board telling the students information; rather he created an arena for discussion and negotiation. During the course of this discussion, the students encountered the need for certain parameters, such as 36 fences must always be used and irregular shapes are not allowed. Jim did not tell the students these constraints at the beginning, but waited for them to be raised by the students. At the end of the class discussion, Jim told the students that it was not enough to write the problem out in his words; they needed to reformulate it in their own words using their own thoughts. More important, Jim did not give them a closed question to answer; he just said, "I am interested in area. I'd like you to explore these shapes and find areas." When Matt said that he was not familiar with the shapes, Jim suggested a place that he could find out about them. He refrained from telling Matt the exact information he needed to know.

The introduction to this problem also illustrates the ways in which teachers introduced structure and guidance. At a number of points, Jim guides the students, asking, for example, whether they *could* have a 36-sided pentagon, to introduce the idea of regular and irregular shapes. Jim introduces algebraic notation to the discussion and tells students about "two ways" of looking at division. Jim deftly weaves mathematical ideas into the conversation about fences, navigating students through the mathematical terrain, drawing from the students' own comments and questions whenever possible. It is characteristic of the Phoenix Park teaching that Jim presented a problem, from which the students generated questions and through which mathematical ideas and methods were introduced. Some of the Phoenix Park problems, such as this one, were contextualized with "real-life" references, others were not, but they were all open enough to encourage different ways of thinking about the mathematics that was or could be entailed.

When the students started their work, Jim left them to their own devices. He did not "police" the room or check that they were going about things in a specific way. When he could, he interacted with students and engaged them in conversations about their work. When one of the students said that a triangular area was bigger than a rectangular area, Jim did not correct him. He asked him whether he would expect this, whether it looked bigger—he encouraged him to think about the situation. It was also typical that students completed differing amounts of work in the remainder of the lesson. Some copied Jim's introduction or another student's introduction in a fairly absent-minded way and did nothing else. Some started their work in a relaxed way, interspersing it with non-mathematical conversations; others engaged in lively debates about the problem. By the end of the first lesson, the students had produced different amounts of work that focused on different questions and problems. As time went by and more lessons were spent on the theme, the students began to diverge more and more, both in the amount of work they completed and the topics on which they worked.

Teachers introduced activities to students which they knew were mathematically rich, but the teachers did not have fixed ideas about the ways in which students would interact with the problems. In a Year 10 lesson taught by Martin Collins, Shelley was working on an investigation into the patterns that emerged from the manipulation of different sets of numbers. After investigating the patterns deriving from combinations of four numbers, Shelly moved on to investigate five:

> After working on the problem for a while, Shelley takes it over to Martin to show him. He looks at the work, laughs, and says, "golly, I didn't know it could get that complicated." Shelley says, "shall I stop?" Martin says, "no, carry on." Shelley says, "I want to carry on because I want to see what hap-

pens to the horizontals when I continue up in this direction." (Martin Collins, Year 10)

This extract is interesting not because Shelley was extending the activity in an unusual or idiosyncratic way, but because Martin had obviously not encountered the particular extensions before. Shelley also demonstrated in this extract that it was the unknown aspect of the exploration that held her interest. She was genuinely interested to know what the mathematical outcome would be of extending the work. When students showed the teachers their work, they did not seem to expect the teachers to have seen it before. They did not expect them to look and say, "yes that is right," but to look and see whether they were moving in an interesting direction. Such interactions then formed the basis for dialogue between students and teachers.

The mathematical content encountered by students during their time at Phoenix Park generally emerged from the projects on which they were working. Ideas within algebra, geometry, number, and data handling would repeatedly recur in relation to each other. The teachers would occasionally stop the class and teach them all a method or ask students to share the directions of their work, but generally the students learned about methods as they became important to the particular investigation they were pursuing. In addition, students received multiple opportunities to develop and use mathematical processes. Indeed the department's approach was designed to integrate aspects of the "using and applying" strand of the National Curriculum, which sets out processes of "application, communication, reasoning, logic, and proof" into every activity the students would meet. Over time, the students became aware of the processes they were learning:

A: It's structured so that . . . it helps with other subjects like science, the results and drawing conclusions, it helps develop those skills. (Alex, Phoenix Park, Year 10, JC)

The students gave other indications that they regarded their mathematics learning as an open experience. In interviews in Year 9, I asked the students to say whether they thought mathematics lessons were similar to any other lessons at the school. Sixteen of the 20 students said that mathematics was most similar to art, English, or humanities; nobody compared mathematics to the subjects more traditionally linked to it, such as science.

JB: Is maths similar to any other lessons at Phoenix Park, or is it different?

L: I suppose it's a bit like English and art and stuff, English, when you're left to do your own work—they explain at the beginning what to do and then you're left on your own to do it. (Lindsey, PP, Year 10, JC)

Differentiated Opportunities

The openness of the activities teachers chose to use at Phoenix Park enabled the provision of differentiated opportunities. This was important to the teachers because they strongly believed that all students should be encouraged to reach high mathematical levels and that students should not be taught in low-level groups. Teachers chose the activities carefully so that they would provide different access points for different students and enable students to work on them at different mathematical levels. The 36 fences activity enabled students to consider the areas of different shapes, draw graphs of relationships, explore combinatorial geometry, learn about trigonometry, and so on. When students began at the school in Year 8, they worked on an investigation called *consecutive numbers*. The investigation asked students to choose three consecutive numbers, square the middle numbers, and multiply the outer ones. In this early lesson, some of the students worked only with different sets of numbers; other students represented the consecutive numbers algebraically. It was commonplace at Phoenix Park for students to engage in mathematics at a variety of levels of difficulty. In one of the activities, students were asked to investigate loci. They were introduced to this concept by going into the playground and being asked to stand in different configurations. At first students were asked to stand 5 meters away from a particular student, then 5 meters from a particular line. They then had to stand at equal distance from two different points. After this introduction to the ideas, students continued to investigate relationships back in the classroom. As a homework for this activity, students were asked to imagine the path of a dot drawn on a circular piece of card that is rolled along a flat surface, then a dot on a triangle, a square, and a shape of their own. Students were told to vary the position of the dot and consider the paths formed. For some students, this was an opportunity to think about shapes and symmetry; for others, it was their first opportunity to learn about Pythagoras; for still others, it was an opportunity to learn about the major and minor axes of a parabola. This activity, like all of those introduced at Phoenix Park, enabled students to move in a number of directions around the mathematical terrain.

In the examination system used in England and Wales, students are entered for one of three levels of the same examination: higher, intermediate, or foundation. The different levels share some of the same questions, but the higher level examinations also include more difficult questions. The different levels give access to different grades. At Amber Hill, the students' examination levels were almost completely determined by the set in which they were located because the different sets were geared toward different levels of mathematics content. This meant that the students' access to different examination grades was partly determined when they were 12 or 13 years of age—3 years before the examinations. At Phoenix Park, the teachers waited to make the decision about different examination levels as late as they could to ensure that all students received opportunity to work toward the highest levels. They made decisions about examination levels when they were required to send lists of students to the examination board toward the end of the students' final year.

Schools have the option of choosing an examination option in which 20% of the students' overall grade is determined by a coursework project completed in school and graded by examination officials. Both schools chose this option. At Amber Hill, the students worked on coursework projects specifically completed for the examination. These took place during 3 weeks of Years 9 and 10, and the students enjoyed them very much. At Phoenix Park, the students and teachers chose projects from those they had worked on over the year. When the students entered their final year and were more aware of examination grades and the ways their coursework contributed to those, the teachers would tell them about the potential of different projects, saying things like, "This is an A/B-ish project and this is a C/D-ish project." Students would use this information to guide their decisions.

There is not space in this book to give many examples of the different projects the students encountered or the different mathematical opportunities the projects provided, but all of the projects shared the characteristic of being sufficiently open to enable different levels of mathematical investigation. If students finished projects or became bored with their work, the teachers would invent extensions for them or offer another idea for students to work on. The following extract, taken from a leaflet prepared by the mathematics department, demonstrates the centrality of the teachers' commitment to mixed ability, differentiated teaching, to their departmental mission:

> Mathematics is a world of powerful and beautiful structures, a way of thinking, organising, investigating and solving problems. It is also, of course, useful in everyday life.
>
> We use a wide variety of activities; practical tasks, problems to solve, investigational work, cross-curricular projects, textbooks, classwork and groupwork. Every task can be tackled by students with widely differing backgrounds of knowledge but the direction and level of learning are decided by

> the student and the teacher. The tasks are chosen so that each student is challenged and stretched at an appropriate level.

Phoenix Park students experienced the freedom to encounter different mathematical ideas and content levels. This meant that some students worked on high-level mathematical topics, such as calculus, that normally would not be encountered until Years 11 and 12. The teachers encouraged such high-level investigations and supported students by holding conversations with them, introducing them to new ideas, and sometimes referring them to books and other reference materials.

Learning to Learn

An important feature of Phoenix Park's approach was the careful attention teachers paid to the way students needed to learn. Corbett and Wilson (1995) argue that those working to promote educational reforms have generally overlooked the fact that students not only need to develop new ways of working in reform-oriented classrooms, but an understanding of and commitment toward the changes in their roles. They argue that, "students must change during reform, not just as a consequence of it" (p. 12). This is a simple but important point that has been given surprisingly little attention. Thus, many teachers have introduced new methods to students, such as working on open problems or having class discussions, without teaching students how to *engage* in these ways of working and how to succeed. Further, there is evidence that knowing how to succeed in relation to new and reform methods of teaching may be a form of knowledge that is inequitably distributed (Jackson, 1989; Lubienski, 2000; Pope, 1999; Zevenbergen, 1996), making it important for teachers to attend to the distribution of such knowledge. David Cohen and Deborah Ball (2000) term the different practices that students need to employ and understand in school *learning practices*. The Phoenix Park teachers seemed to pay careful attention to the learning practices that students needed to develop, teaching students how to learn in an open approach.

When students began at Phoenix Park, they had encountered more closed and traditional presentations of mathematics for the previous 8 years. The students reported that they had to make a number of adjustments when they arrived at their new school:

> A: It was a big change from my last school, having the books and then just having it written on the board and being told to get on with it. (Andy, PP, Year 9, RT)

Many of the students took to the approach with ease, reporting that they were appreciative of the opportunity to work in more open ways. But some of the students, particularly boys, found the openness of the work extremely disconcerting. They said they were uncomfortable with the lack of structure or suggested direction in the problems they met, and that they preferred a more traditional approach. These students, along with the majority of students at Phoenix Park, came from homes of severe poverty, living on a housing estate (similar to a U.S. project), where police would not venture at night. When I interviewed the students at the beginning of the study, they described their motivations clearly:

S: When I go into a maths lesson I usually sit down and I think, who am I going to throw a rubber *(erasure)* at today? (Shaun, PP, Year 8, RT)

JB: Can you think of a maths lesson that you've enjoyed?
M: Messing about, that's what I enjoy doing.
JB: What would make maths better?
M: Working from books—you don't mess about if you've got a book there, you know what to do. (Megan, PP, Year 8, RT)

Although some students *blamed* their misbehavior on the openness of the work, the teachers did not give the students books or structure. This may have been the easiest option, but the Phoenix Park teachers believed that the open-ended approach they used was valuable for *all* students and that it was their job to make the work equitably accessible. They therefore developed a range of practices that served to increase the students' access to the problems they met and the methods they were expected to use.

One practice that was central to the Phoenix Park teachers' approach was that of introducing the activities to students themselves, which enabled the teachers to decide on the degree of support or structure students needed. In the 3 years that students attended Phoenix Park, they were never left to interpret text-based problems alone. The teachers always spent time with individuals, groups, or the whole class introducing ideas and making sure the students all knew how to start their explorations. At Phoenix Park, the teachers would frequently ask the students to gather around the board before leaving the class so they could all have some discussion of the homework problems being posed. M. Smith (personal communication, 2001) reported her observations of a middle-school teacher in an urban school in the United States who used a reform curriculum. She reported that the teacher would ask students to read problems aloud in class and then hold a discussion about the context of the problem and any vocabulary used in case either was unfamiliar.

Then she would ask students to discuss what they thought the task was asking them to do. After such discussions, the teacher would ask groups to work on the task and check that different individuals understood what they should do. Such practices were also employed by Phoenix Park teachers in order to make tasks equally accessible, but they contrast with many classrooms I have visited in which students are left to interpret the aim of problems from their reading of reform texts, which are often extremely wordy and linguistically demanding. The way in which work is introduced to students and the access students are given to the mathematical ideas that they are intended to explore seems extremely important for the attainment of equity.

A second important feature of the Phoenix Park teachers' practice was that they paid attention to the ways in which students communicated their understanding, as well as the students' understanding of the need for that aspect of their work. In more traditional mathematics classrooms, such as those of Amber Hill, students are required to produce correct answers. In reform-oriented classrooms, students often need to go beyond correct answers and explain their methods, justifying the approaches they have used. At Phoenix Park, the teachers paid careful attention to this aspect of the students' role and helped them understand the particular learning practices in which they needed to engage. For example, in one of the lessons I observed, the teacher asked all the students to gather round the board; then she posed the following question: "If someone new came into class and asked you what makes a good piece of work, what does Ms. Thomas like, what would you say?" The first student offered "lots of writing"; others offered suggestions such as, "have an aim," "draw a plan," and "write about patterns."

Each time the teacher came back with further questions—such as "is the amount of writing important?" "what does that mean?" "why is a plan important?", "what does a good plan look like?" "why do we record patterns?" The students struggled over many of their explanations, but they sat around the board engrossed in this discussion for some time. The students were clearly appreciative of the opportunity to learn about valued ways of working. As they talked, the teacher kept a record of the students' suggestions on the board. After approximately 40 minutes of discussion, the teacher told the students that their task was to design a poster describing the different features of "good work." She also gave them a page that the department had prepared called *hints for investigations*. It was divided into three columns headed *what to say, how to say it,* and *making sense of it.* These showed different suggestions for students, such as, "Can you make the problem more general?", "Make the original problem more difficult," and "Now explain how or why your algebraic rules work." The students studied the page and incorporated many of the suggestions into their posters. This lesson explicitly focused on the mathematical learning practices (Cohen & Ball, 2000) that the students needed to employ in the pursuit of their mathematical investigations.

The Phoenix Park teachers frequently encouraged individual students to explain their reasoning and communicate in more detail because the students were not used to doing so when they arrived at the school. In one of the lessons I observed, a student gave a problem on which he had been working, which showed some of his methods and a correct answer. The teacher studied it for a while and then said:

> Brilliant work John but you can't just write it down, there must be some sense to why you've done it, some logic. Why did you do it that way? Explain it. (Rosie Thomas, Year 10)

Rosie's "there must be some sense to why you've done it" typifies the sort of encouragement the students were given at Phoenix Park. The teachers strove to expand the way in which the students thought about mathematics, extending the students' value systems beyond the desire to attain correct answers. The teachers at Phoenix Park developed a range of practices aimed at helping all students understand what they needed to do to function successfully in a reform classroom, and there was considerable evidence that they were successful in that regard:

I: It's an easier way to learn, because you're actually finding things out for yourself, not looking for things in the textbook.

JB: Was that the same in your last school do you think?

I: No, like if we got an answer, they would say, "you got it right." Here you have to explain how you got it.

JB: What do you think about that?

I: I think it helps you. (Ian, Phoenix Park, Year 9)

When the Phoenix Park teachers found that some students were not communicating their thinking or interpreting numerical answers, they devoted more time to this aspect of their teaching, regarding the students' reluctance as a gap in their understanding of what was required in the work. After months of careful support from the teachers, the reluctant students started to become more engaged with their work and more comfortable with the freedom they were given. The change in some of the disaffected boys became most obvious when, in the second year of the school, they were taught by a student teacher who tried to teach mathematics in a more traditional way. In the following extract from my observation notes, the boys start to complain because of the *closed* nature of the work given to them. This was very different from the approach to which, by then, they had become accustomed:

> The student teacher starts the lesson by asking the class to copy what he is writing on the board. He is writing about different forms of data, qualitative and quantitative. The students are very quiet and they start to copy off the board. The teacher then stops writing for a while and tells the students about the different types of data. He then asks them to continue copying off the board. After a few minutes of silent copying, Gary shouts out, "Sir when are we going to do some work?" Leigh follows this up with, "Yeah are we going to do any work today sir?" Barry then adds, "This is boring, it's just copying." The teacher ignores this and carries on writing and talking about data. The boys go back to copying. The teacher looks across at Lorraine, who is looking puzzled, and asks her if she "is OK." She says, "No not really, what does all this stuff mean?" This seems to annoy the teacher or make him uncomfortable; he turns back to the board and continues writing. Gary persists with his questioning, this time asking, "Sir, why are we doing all this?" The teacher replies: "We are just rounding off the work you have done."
>
> After about 20 minutes of board work, the teacher asks the students to go through all of their examples of data collection that they have done over recent weeks and write down whether they are qualitative or quantitative. Peter asks, "Sir what's the point of this? Aren't we going to do any work today?" the teacher responds with, "You need to know what these words mean." Peter replies, "But we know what they mean, you've just written it on the board so we know." (Phoenix Park, student teacher, Year 9)

This series of interactions was particularly interesting to observe because it was the group of boys who had been most resistant to open-ended work when they started at Phoenix Park who objected to the closed nature of the work the student teacher gave them. The boys repeatedly asked "whether they were going to do any work today," indicating that they did not regard copying off the board as work probably because it did not present them with a problem to solve. When the student teacher told them to classify data as quantitative or qualitative so that they would learn what the words meant, Peter questioned the point of this because they had already been told what they meant. Yet the mathematics teaching offered in this example is fairly characteristic of more traditional high school mathematics pedagogy, in which the teacher explains what something means to students, they copy it down from the board, and then they practice some examples of their own. The degree of resistance the students provided seems important to consider partly because it gives an indication of the ways students adapted to their school approach. Teachers often talk of students' resistance to the use of open problems or the need to discuss methods, but this interaction showed that even the most reluctant students changed their expectations for ways of working with careful support. In one week that I was in school, Jim had been absent for one lesson and the class had been taught by a substitute teacher from outside the department. When he returned, one of the previously resistant boys complained about the substitute teacher, saying to Jim: "It was terrible—we had this teacher who acted like he knew all the answers and we just had to find them."

The Phoenix Park teachers paid explicit attention to the learning practices students needed, teaching students *how to learn* as well as teaching them mathematics. Many of the practices were those that are valued in other reform-oriented classrooms, but teachers do not always give them such explicit attention. They assume that students will understand the need for their use and the changed practices they need to employ.

Time on Task

Another striking aspect of school mathematics at Phoenix Park related to the number of students choosing not to work in lessons, which continued to be a source of surprise to me. In their Year 9 questionnaire, students were asked to describe their mathematics lessons to someone from another school. The most popular description from 23% of students was "noisy." In the 100 or so lessons I observed at Phoenix Park, I would typically see approximately one third of students wandering around the room chatting about nonwork issues and generally not attending to the project they had been given. In some lessons, and for some parts of lessons, the numbers off task would be greater than this. Some students remained off task for long periods of time, sometimes all of the lessons; other students drifted on and off task at various points in the lessons. In a small quantitative assessment of time on task, I stood at the back of lessons and counted the number of students who appeared to be working 10 minutes into the lesson, halfway through the lesson, and 10 minutes before the end of the lesson. Over 11 lessons, with approximately 28 students in each, 69%, 64%, and 58% of students were on task, respectively.

The freedom that the students experienced to stop working when they wanted to seemed to be created by a number of interrelated facets of the Phoenix Park approach. It was partly to do with the nature of the mathematical approach and the fact that students could be wandering around the room and chatting with other students as part of their work. It also related to the fact that the students could all have been working on something different, which made it difficult for teachers to monitor the amount of work that they completed:

T: It gives some people more of a chance to muck about *(misbehave).*

JB: Why?

T: Because, for instance, at the end of a lesson if the teacher wanted to check how much work you'd done he couldn't, but if you started at number 1 he would know that you hadn't got to number 20 or whatever. (Trevor, PP, Year 10, RT)

More important than either of these factors, however, is that the freedom the students experienced seemed to relate directly to the relaxed and nondisciplinarian nature of the three teachers and the school as a whole. Most of the time, the teachers did not seem to notice when students stopped working unless they became very disruptive. All three teachers seemed concerned to help and support students and, consequently, spent almost all of their time helping students who wanted help, leaving the others to their own devices. The three teachers were not markedly different in this regard, although Jim Cresswell's lessons were noticeably more chaotic than those of the other two teachers.

> I think the weakness of my teaching style would be very much that I depend on willingness and co-operation and, you know, if somebody is motivated to do the stuff they will achieve well. (Jim Cresswell)

Jim often told me that he was "no good at discipline," and my lesson observations showed that students in his classes were less on task than the classes of other teachers. This was partly because he treated the students in an adult way and some of the students took advantage of this. For example, there was a small classroom attached to Jim's room that nobody used. Jim used this room as a talking room, and students were meant to work in there if they wanted to talk and work, leaving the other students to work in quieter conditions. Jim was not concerned about his inability to see the students in this room, and he rarely asked students to work when they were not doing so unless they became disruptive. When Jim did tell students to work, the result was often ineffective. Typically the students would say something back to Jim, which sparked a debate between Jim and the student. At the end of this, the student usually went back to not working, and Jim would usually be called away to help somebody. In a number of Jim's lessons I observed, so few of the students appeared to be working that I started to have serious doubts about my research study. At the end of my research, I found out that some of the newer, more middle-class parents at the school had complained about Jim's teaching, which resulted in the principal visiting his lessons and telling Jim that only about 30% of students were on task.

Both Rosie Thomas and Martin Collins showed more overt concern to keep students on task than Jim. But while both teachers were more likely to react to the extremes of behavior that Jim tolerated, they nevertheless seemed unconcerned about students who sat and chatted through parts of their mathematics lessons. When the two teachers asked students to work, this often had little effect: The students worked for a few minutes, then went back to chatting. The degree to which students were on task in lessons also varied between classes, year groups, and aspects of lessons. In later sections, I explore the impact of the independence that students experienced over their work rate, which produced some surprising results.

Independence and Choice

There were many overt and covert ways in which the students at Phoenix Park were encouraged to be independent. This meant that they needed to take on some responsibilities as part of their mathematics approach in order to succeed. For example, the students were not given exercise books for their work: They used pieces of paper. At the start of activities, they were given blank or lined pieces of paper as well as graph paper if they needed it. The students each had a box file in which they kept their work. Nobody took charge of this process for the students; papers were not collected at the end of lessons. Students were meant to either take them home and bring them back again or store them in their box file. Students often came to lessons having forgotten or lost their work from the previous lesson and so took a new piece of paper and continued on that. Some of the students were disorganized, and their box files were made up of odd collections of extracts from different activities. At the end of each project, students were meant to gather together their work, present it in a coherent fashion, and summarize it. The students were rarely encouraged to be careful or tidy, and many of the finished projects looked messy compared with a more typical mathematics exercise book.

At around Easter of Year 10, the school sent pieces of coursework to the official examination offices. At Amber Hill, the projects were the only ones worked on for 3 weeks of Years 9 and 10. At Phoenix Park, the teachers and students could select the best projects from all those they had worked on. But the teachers gave the choice to the students, who were told to choose their best two pieces of work and give them in. The teachers gave the students guidance if asked. Often the pieces of work that were sent to the examination officials were unfinished either because the students showed little concern for the task of choosing their coursework or because the students had no complete projects to send:

L: They left it to the last minute as well, like they kept saying you've got to have work for your GCSE and that, but if you didn't hand your projects in, in years 8 and 9 they weren't really bothered were they?

H: No.

L: And at the end now they say we need them. (Louise and Hannah, PP, Year 10, JC)

Here the students related the incompleteness of their work to the lack of enforced discipline or control from their teachers. It seemed surprising to me that the teachers gave such an important responsibility to the students, but this was consistent with the general approach of the school:

P: The amount you do is always up to you isn't it? How much homework you do and especially course-work for GCSE, it's your work, it's your responsibility, I mean however much work you get in, that's always going to be reflected in your mark. (Nile, PP, Year 10, JC)

Another important responsibility that the students held emanated from the choice that students were given about the projects they could work on and the direction in which the students took their work. The students at Phoenix Park were given considerable and varying amounts of freedom in their choice of work, their approach to work, the way in which they behaved in lessons, the organization of their work, and even their work environment. This choice and the students' independence had an important impact on their responses to mathematics, which are considered next.

THE STUDENTS' RESPONSES

Enjoyment

In interviews, conversations, and lesson observations at Phoenix Park, the students gave a much more varied picture of their enjoyment than the students at Amber Hill. The Amber Hill approach prompted a fairly consistent reaction from the students, whereas the Phoenix Park approach seemed to divide the year group into those who loved it, those who liked it, and those who hated it.

Approximately half of the Phoenix Park cohort liked mathematics most of the time, but their enjoyment depended on the particular projects they were doing.

Approximately one third of the students was more positive than this, and they seemed to like everything about mathematics. Questionnaires and interviews in Years 8 to 10 showed that these students liked the approach because it was varied, they were given a choice about what they did, and they had the freedom to work in any direction.

V: I thought the activities were really interesting because you had to work out for yourself what was going on, you had to use your own ideas.
JB: How does that compare to the SMP work you used to do in middle school?
V: Boring, it was boring doing stuff out of books. (Vicky, PP, Year 10, JC)

S: You're able to explore more, there's not many limits and that's more interesting. (Simon, PP, Year 10, JC)

However, this freedom was also the reason the third group of students hated the approach. Approximately one fifth of the cohort thought that mathematics was too open, and they did not want to be left to make their own decisions about their work. They complained that they were often left on their own not knowing what to do, and they wanted more help and structure from their teachers. The students felt that the school's approach placed too great a demand on them—they did not want to use their own ideas or structure their own work, and they said that they would have preferred to work from books. What for some students meant freedom and opportunity, for others meant insecurity and hard work. There were approximately five students in each class who disliked and resisted the open nature of their work. These students were mainly boys and were often disruptive—not only in mathematics, but across the school.

In the Year 8 questionnaire item that asked students to describe the *most interesting piece of mathematics* they had ever done in a lesson the Phoenix Park, students responded in a different way than the Amber Hill students. Whereas 49% of Amber Hill students chose *Logo*, Phoenix Park students described a variety of different projects. Five different projects were nominated by at least 5% of students: 11% of students chose *Logo*, 10% an activity called frogs, 9% a probability project, 8% the maths day (when they worked on mathematics projects all day), and 6% an activity called *limping seagulls*. Another 36% of students chose other class projects encountered over the past year. The question asked students to describe the most interesting piece of mathematics they had ever done in a school lesson. Many of the Amber Hill students described a lesson from elementary school or from Years 6 and 7. At Phoenix Park, all of the students described one of the projects they had experienced since starting at Phoenix Park in Year 8, and all descriptions were positive. For example:

"Horse racing was good because the answers were unexpected."

"The best piece of maths I think I have done was boxes as I did quite a long project."

"Statistics, I thought this was the most interesting, I wrote a large amount about marriages and divorces using the book *Social Trends.*"

The Phoenix Park students' replies gave the impression that they were genuinely interested in the projects they had chosen, and they did not report that mathematics lessons were monotonous or boring.

In Year 8, students from both schools were asked in a questionnaire to state how often they enjoyed the mathematics they did in school (*always, most of the time, sometimes, hardly ever,* or *never*). This closed question produced similar results from the two schools. Forty-three percent of Amber Hill students and 52% of Phoenix Park students reported enjoying mathematics *always* or *most of the time.* However, the students responded very differently to open questions on the same questionnaire. One question asked the students to describe what they disliked about mathematics at school. Forty-four percent of Amber Hill students strongly criticized the mathematics approach, and 64% of these students criticized the textbook system. At Phoenix Park, 14% of students criticized the school's approach, and the most common response—from 23% of students—was to list nothing they disliked about mathematics at school. This compared with 6% of Amber Hill students. Table 5.1 presents all of the responses the students gave to the three different open questions on the questionnaire, which asked students what they liked, disliked, and would like to change about mathematics lessons. These three questions prompted 382 comments from the 160 Amber Hill students and 202 comments from the 103 Phoenix Park

TABLE 5.1
Year 8 Open Questionnaire Responses

Nature of Comment	*Amber Hill* % (*n* = 382)	*Phoenix Park* % (*n* = 202)
Enjoy open-ended work	14	38
Dislike textbook work	22	0
Cannot understand work	20	6
Can understand work	3	5
Work is interesting	4	21
Want more interesting work	15	19
Want more group work	5	0
Enjoy working alone/with others	8	4
Pace is too fast	9	3
Pace is about right	0	3

students (there were no significant differences between the number of comments per student at the two schools). The responses have been combined to present an overview of the issues important to the students.

This table shows that students at the two schools chose to address different issues when they were invited to give their own opinions on mathematics lessons. The Phoenix Park students chose to comment on the interest of their lessons and their enjoyment of open-ended work. These two sentiments comprised 59% of all the Phoenix Park comments. The Amber Hill students were more concerned about lack of understanding and their dislike of textbooks; these two comments comprised 42% of the Amber Hill responses. Many more of the Amber Hill students probably would have talked about open-ended work if they had ever experienced any, but at that time they had not yet worked on their coursework projects. In response to the three questions, there were 88 comments (23%) from Amber Hill students that related to their perceived lack of understanding of mathematics, compared with 6 comments (3%) from the Phoenix Park students.

In their Year 8 questionnaire, the students from both schools were asked to write a sentence describing their mathematics lessons. The three most popular descriptions from the 75 respondents at Phoenix Park were *noisy* (23%), *a good atmosphere* (17%), and *interesting* (15%). This contrasted with the three most popular responses from the 163 Amber Hill respondents, which were *difficult* (40%), *something related to their teacher* (36%), and *boring* (28%). The students' sentences were also coded as either *very positive, positive, neutral, negative,* or *very negative.* Table 5.2 shows the distribution of results for the two schools.

The overall picture of enjoyment gained from Phoenix Park was therefore more varied and significantly more positive than that gained from Amber Hill. At Phoenix Park, the vast majority of disaffection was suggested by a small proportion of students who showed opposition to school in general. A consideration of the various forms of data, including questionnaires, interviews, and lesson observations, suggests that approximately one third of the Phoenix Park students positively liked mathematics particularly because of its variety and openness, approximately one half of students enjoyed some of the projects some of the time and disliked others at other times, and the remaining students disliked the approach,

TABLE 5.2
Describe Maths Lessons: Coded Responses

%	*very positive*	*positive*	*neutral*	*negative*	*very negative*	*n*
AH	0	23	38	33	6	154
PP	5	38	32	25	0	67

particularly the freedom and openness they experienced. I consider this last group of students in more depth in a later part of this chapter.

Engagement

The General Picture. The Phoenix Park students varied in the extent to which they engaged with their mathematics. They were often left to decide whether or not they worked in class. This meant that some students worked with enthusiasm on their mathematics projects, while others would find talking or disrupting the class more interesting than their work. It was difficult for the students to work in a procedural way at Phoenix Park because the students constantly needed to make decisions about their project work. This meant that students tended to be either interested and working or uninterested and not working. The following extract is taken from the third lesson on the theme *36 fences*, which was described earlier and taught by Martin Collins. Some of the students have considered the areas of different rectangles with a perimeter of 36; others have moved on from this and have started to investigate the areas of different shapes.

> Mickey has found that the biggest area for a rectangle with perimeter 36 is 9 × 9 and is moving on to find the area of equilateral triangles, compared with other triangles; he seems very interested by his work. He finds one area and is about to find another when he is distracted by Ahmed, who tells him to forget triangles, he has found that the shape with the largest area made of 36 fences has 36 sides. He tells Mickey to find the area of a 36-sided shape too and leans across the table explaining how to do this excitedly. He explains that you divide the 36-sided shape into triangles and all of the triangles must have a 1cm base. Mickey joins in saying, "yes and their angles must be 10 degrees!" Ahmed says, "yes but you have to find the height and to do that you need the tan button on your calculator, T-A-N, I'll show you how. Mr Collins has just shown me." Mickey and Ahmed move closer together to do this. At another table, I ask Clare what she is doing; she says that she is working out the area of a hexagon and she shows me her diagram. She explains that she is working out the area by dividing it into six triangles; she has drawn one of the triangles separately. She says that she knows that the angle at the top must be 60 so she can draw it exactly to scale using compasses and find the area by measuring the height. Clare seems to have made these decisions on her own and she is clearly interested in her work. At another table, six girls have not started work even though we are 20 minutes into the lesson; they are sitting coloring on their folders; another group of boys are working out the areas of rectangles, but they do not seem to be particularly interested in what they are doing. (Year 8, Martin Collins)

This extract demonstrates the different amounts of enthusiasm and interest that were commonly in evidence during Phoenix Park lessons. Clare was not a high-attaining student, but she was interested in what she was doing and the decisions she had made. The six girls who were drawing on their folders were clearly not interested, and the small group of boys working out the areas of rectangles were not working with enthusiasm. Mickey and Ahmed were two high-attaining boys who were extremely involved in their work and who seemed genuinely excited to be discovering some new mathematical methods and relationships. The interest they showed for trigonometry, because they could *use* a tangent ratio to help them find something out within their project, was vastly different from the interest the Amber Hill students showed toward trigonometry. Students at both schools learned trigonometry within contextualized questions regarding shapes, but the Amber Hill students were introduced to trigonometric rations, then required to practice them within contextualized questions. The Phoenix Park students met trigonometric ratios when they needed them to solve problems. The students at Phoenix Park seemed to regard trigonometric ratios as exciting, reminding me of this description that Margaret Wertheim (1997) offers of the way she was introduced to pi when she was a child:

> When I was ten years old I had what can only be described as a mystical experience. It came during a maths class. We were learning about circles, and to his eternal credit our teacher, Mr Marshall, let us discover for ourselves the secret of this unique shape: the number known as pi. Almost everything you want to say about circles can be said in terms of pi, and it seemed to me in my childhood innocence that a great treasure of the universe had been revealed. Everywhere I looked I saw circles, and at the heart of every one of them was this mysterious number . . . It was as if someone had lifted a veil and shown me a glimpse of a marvelous realm beyond the one I experienced with my senses. (p. 3)

The students at Phoenix Park responded in similar ways to a number of their mathematical discoveries. This means that they experienced moments of wonder and excitement for at least *some* of their mathematics careers, which contrasted strongly with the Amber Hill students.

Another important difference between Amber Hill and Phoenix Park was that Phoenix Park students were not made to work. In interviews, the students did not talk about work they had done because they had been forced to but had gained little from in the way that the Amber Hill students did. They talked instead about the choice they had between involvement and doing nothing:

H: It was definitely a lighter lesson—you'd be involved and if you didn't want to be involved you'd sort of sit back and watch it all happen I suppose. (Hannah, PP, Year 10, JC)

Here Hannah does not give working without involvement as an option, whereas this was something of which the Amber Hill students were acutely aware. Although the freedom that the students at Phoenix Park experienced over their work rate meant that some students did very little, it also meant that some students worked in a motivated way. When students were asked to say the amount of time they worked in lessons, the results were interesting. Figure 5.1 presents the Phoenix Park students' results alongside those from Amber Hill to demonstrate the difference in the distribution of the results. At Phoenix Park, the students' times produced a symmetrical distribution, indicating that when students were given the freedom to work (or not), some students did very little work, but as many chose to do a lot. Indeed a much greater proportion of Phoenix Park students reported working for 51 to 60 minutes than Amber Hill students, who were made to work (12% at Phoenix Park, 2% at Amber Hill). Despite these differences, the means of the times given by Amber Hill and Phoenix Park students were identical (37 minutes). In some senses, this is remarkable given the difference in the freedom experienced by the two sets of students. Earlier I described the relaxed nature of Jim Cresswell's lessons and said that these lessons appeared to be more chaotic than those of Martin and Rosie. However, the means of the times given by students of the three teachers at Phoenix Park were as follows: Rosie, 40 minutes; Jim, 39 minutes; and Martin, 32 minutes. Martin was reported by the students to be the strictest of the three teachers. The similarity between the times given by students of different teachers and the times of students at the different schools adds further weight to the idea that making students work is not a particularly effective way to get students to think about mathematics.

The Uninterested Students. In every mathematics lesson I observed at Phoenix Park, between three and six students would do little work and spend much of their time disrupting others. I now try to describe the motivation of these 20 or so students, who represented a small but interesting group. The students who did little work in class were mainly boys, and they related their lack of motivation to the openness of the mathematical approach and, more specifically, the fact that they were often left to work out what they had to do on their own.

S: I tend to doss about a lot in maths *(mis-behave)*, half the time I can't be bothered to call miss over or ask her what I want to know, but I do realize that maths GCSE is pretty important.

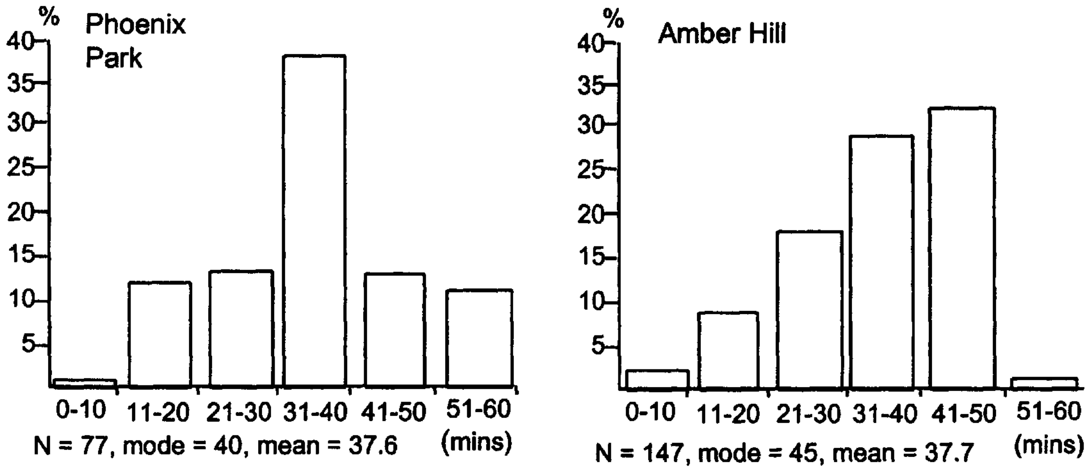


FIG. 5.1. Students' perceptions of time spent working in 60-minute mathematics lessons at Phoenix Park and Amber Hill.

JB: Why do you mess about in maths more than other subjects?

S: Because half the time if I ask for help I don't get it, or I don't get it until 20 minutes after I've asked. (Shaun, PP, Year 10, RT)

Many of the Phoenix Park students talked about the difficulty they experienced when they first started at the school working on open projects that required them to think for themselves. But most of the students gradually adapted to this demand, whereas the disruptive students continued to resist it. In Years 9 and 10, I interviewed six of the most disruptive and badly behaved students in the year group: five boys and one girl. They explained their misbehavior during lessons in terms of the lack of structure or direction they were given and, related to this, the need for more teacher help. These students had been given the same starting points as everybody else, but for some reason seemed unwilling to think of ways to work on the activities without the teacher telling them what to do. This was a necessary requirement with the Phoenix Park approach because it was impossible for all of the students to be supported by the teacher when they needed to make decisions.

The students who did not work in lessons were no less able than other students; they did not come from the same middle school and they were socioeconomically diverse. In questionnaires, the students did not respond differently from other students, even on questions designed to assess learning style preferences. The only aspect that seemed to unite the students was their behavior and the fact that most of them were boys. The reasons that some students acted in this way and others did not were obviously complex and due to a number of interrelated factors. Martin Collins believed that more of the boys experienced difficulty with the approach because they were less mature and less willing to take responsibility for their own learning than the girls. The idea that the boys were badly behaved because of immaturity was also partly validated by the improvement in the boys' behavior as they got older:

I: We have wasted a lot of time in the lessons, some of it, we have wasted time.

G: Yeah, we didn't used to do any work in lessons at all.

JB: But you take it more seriously now?

G: Yes.

JB: Why?

G: I'm not sure, in maths, then, we used to. . . .

I: Chuck *(throw)* stuff.

G: Yeah we always used to be chucking stuff and fighting, now we're a bit more serious. (Gary & Ian, PP, Year 10, JC)

The misbehaving students in each group were generally street-wise, confident students who seemed to enjoy being the center of attention. It was as if they had decided that school work was not for them, but they could gain satisfaction and self-esteem from being part of an antischool subculture. Other research studies have shown the presence of students with antischool values who gain pleasure from misbehaving (Ball, 1981; Willis, 1977), but the Phoenix Park students experienced more freedom than students generally do in schools. The result of this freedom seemed to be that they did little work. The students were also expected to do a lot in mathematics lessons. They were not asked to work through pages of a book following a rule. Instead they were asked to think for themselves, plan their work, and solve problems. They needed to make decisions and coordinate strategies. For many of the students, who were probably more inclined to "mess about" than work when they arrived at the school, this was too much, as one of the girls who shared lessons with them commented:

H: Well I don't think they were stupid or anything they just didn't want to do the work, they didn't want to find things out for themselves, they would have preferred it from the book, they needed to know straight away sort of thing. (Helen, PP, Year 10, MC)

Although the students at Phoenix Park who did little work in lessons were distinct from other students at the school, their behavior in lessons was only a more extreme version of a behavior displayed by most students at some times during lessons. The students worked when they wanted to work, which, for most students, meant intermittently.

S: But the tables that don't, even the tables that do get on with their work tend to jabber on a bit, like, Miss Thomas goes over to the table and she'll say "Oh did you see Neighbours *(TV Show)* last night?" to the other table and then they'll start talking and everyone will be talking. (Shaun, PP, Year 10, RT)

In summary, the students at Phoenix Park spent less time working than the students at Amber Hill, but they seemed to spend more time engaged with their work. This was not true for all of the students, but the widespread lack of interest evident at Amber Hill was rarely witnessed at Phoenix Park. This was replaced by a much more varied response to work, which, for most students, included both times of involvement and times of nonmathematical activity.

Students' Views About the Nature of Mathematics

The students at Phoenix Park were very different from the students at Amber Hill in the way they viewed mathematics. This was because most of the students believed mathematics to be an active, inquiry-based discipline. In the Year 9 questionnaire item that asked students to prioritize either thought or memory (see chap. 4), 65% of Phoenix Park students chose thought, compared with 36% of Amber Hill students. The majority of the Phoenix Park students did not regard mathematics to be a rule-bound subject involving set methods and procedures that they needed to learn; they saw it as a subject of explorations, negotiations, and inquiry:

A: You explore the different things and they help you in doing that. (Alex, PP, Year 10, JC)

P: You can do it at your own level, what suits you, and it's very sort of open. You can use it in different ways, you can do different things more than with set questions.
S: You're able to explore, there's not many limits and that's more interesting. (Philip & Simon, PP, Year 10, JC)

The students also had a sense of mathematics as a subject that allowed them to think deeply—to go beyond surface features of questions:

P: It's when you like learn new ways of doing things or you're like doing quite well on a problem... you're taking it really far, the investigation, you're getting really deep into it . . . you feel like you're learning quite a lot more. (Philip, PP, Year 10, JC)

There is evidence that many students regard mathematics to be a collection of procedures that allow them to answer questions in a short space of time (Schoenfeld, 1988). The Phoenix Park students did not seem to have this shallow view of mathematics; they were aware of the depth of the subject—the different layers that may be encountered. The students also demonstrated an unusual awareness of the diversity and breadth of mathematics. They did not regard mathematics as a vast collection of "sums"; they seemed to have a richer and more balanced view of the subject:

A: I used to think that maths was just sums and hard work.
JB: Don't you now?
A: No, not really, some of it is, but there's a lot more stuff involved in it as well.
JB: What other stuff?

A: Well, different sorts of—well there's loads of different things, theories and stuff like that, formulas, algebra, shapes and stuff. (Alex, PP, Year 10, JC)

JB: Has doing the projects changed the way you think in any way?

D: Yes 'cause like bookwork—say it's just all sums or whatever, but that's only like one really small part of maths isn't it?

JB: Mmm.

A: If you're doing all problems and that you can learn about all the different areas. All the really advanced maths is a lot more to do with theorems and theories and that sort of thing than just sums. (Danny & Alex, PP, Year 10, JC)

Neither Danny nor Alex particularly liked mathematics compared with their other school subjects, but this did not appear to affect the way in which they constructed their views about the *nature* of mathematics. Both students showed that they regarded mathematics as a diverse subject in which "sums" were "only one really small part." In their Year 8 questionnaire, students were asked to describe one or more situations when they had used mathematics outside school. Seventy-seven percent of the Amber Hill students' comments related to money or shopping, and no descriptions were given of situations requiring the use of data handling, shape, or space. At Phoenix Park, 53% of comments also related to money and shopping, but 14% of students described less typical activities such as: sorting out a magazine collection, classifying option choices at a club, laying slabs in the front garden, organizing a bank account, reading a map, and organizing a route for a paper round. These were not examples that the students had been told about in class or contexts they had encountered in lessons.

In many of my lesson observations, the students approached and talked about mathematics in ways that were qualitatively different from most students I have observed in mathematics classrooms over the last 15 years. They showed that they were not only interested in the answers to the investigations and problems, but were aware of the importance of the methods and processes they used along the way:

P: Sometimes I can't really think how things can be used, but it's the process and the method, I suppose, and the way you look at it. (Philip, PP, Year 10, JC)

The students' awareness of the methods and processes they used in their work can probably be related to the encouragement their teachers gave them to think about methods and strategies and the careful attention teachers paid to ways of working. Students were often asked to think about what they had been doing in lessons and to plan the direction of the rest of their work for homework. These homeworks stand in direct contrast to the more typical "finish up to question 20" mathematics homework. They explicitly required students to think about strategies and methods. There were many indications that the teachers were successful in this regard, and that the unusually dynamic views the students held about mathematics were formed in response to their project-based work. All of the students contrasted this work with the SMP bookwork they encountered in middle school:

H: It's more interesting now, you're not just working through a book doing the same things. (Helen, PP, Year 9, RT)

S: You go right through the pages of a book until you've finished it and then it takes you to other pages, all pretty much the same stuff, you can't really experiment with work in books. (Shaun, PP, Year 9, RT)

L: It gives you more freedom here and it lets you find things out for yourself, where a book would just give you all the answers and stuff and you wouldn't have to find things out for yourself, you have to find things out for yourself and its more interesting and I think you tend to remember it more when you've found things out for yourself. (Louise, PP, Year 10, JC)

The Phoenix Park students had all experienced a book-work approach to mathematics prior to their project-based work, and the contrast they offered between the two approaches focused on the more dynamic nature of the mathematics they encountered in their project work. They talked about the way that books did not give them anything to *find out* or *explore;* they merely gave them *set work* that they had to *work through.* The students highlighted the *procedural* aspect of book work, which, they said, made mathematics less interesting and useful for them.

The significance of the students' project work to the active views of mathematics that they had developed was also demonstrated by the results of their Year 9 questionnaire. At Phoenix Park, the students worked in an entirely open way until Christmas of Year 10, when they started preparing for examinations. At this time, the mathematics approach became considerably more procedural as the students were required to work through short, procedural examination questions. When my case study year group was in Year 9, I gave a questionnaire to students in Years 8, 9, and 10. This asked students to prioritize either thought or memory. Sixty-six percent of students completing Year 8 and 65% of students completing Year 9 thought it was more important to think hard about questions than remember similar questions. This proportion fell to 48% of students completing Year 10. At another point in the questionnaire, the students were asked to rank different areas of mathematics in terms of importance. Five percent of Year 8 students and 8% of Year 9 students thought that "remembering rules and methods" was the most important part of mathematics; in Year 10, this increased to 17% of students. Responses to the same questions given to three year groups at Amber Hill remained constant between the three year groups (17%, 15%, 15%).

The Phoenix Park students' responses to their Year 10 examination preparation indicate that the change from project work to a more formal mathematics approach prompted a corresponding change in the students' views about mathematics. Cobb, Wood, Yackel, and Perlwitz (1992) also found this to be true of students who worked on projects and then reverted to a textbook approach. This caused many more of the students to think that success in mathematics involved following a teacher's set methods. At Phoenix Park, the project-based approach had expanded the students' views of mathematics and caused them to regard mathematics as an active, exploratory discipline; in contrast, the examination work caused many students to go back to some of their old views about the limited nature of mathematics, thus eradicating some of the school's positive achievements.

Independence and Creativity

The students at Phoenix Park were encouraged in many different ways to be independent in mathematics, mainly through the degree of choice they were given and the responsibility they needed to take for their work. In their Year 9 questionnaire, students were asked to describe mathematics lessons, and 11% of students *chose* to comment on the independence they experienced in their lessons. For example, "what you do is mostly up to the pupils." None of the Amber Hill students responded in this way. When teachers at Phoenix Park interacted with students, they treated them as if they were equals. If they asked students to do something and the students asked why, they would explain rather than say, "because I said so." The teachers did not seem to distance themselves from students, and the gap between teachers and students was not distinct. This seemed to have a direct effect on the students. When they interacted with adults, even strangers, they were confident and chatty; they never appeared to be nervous or intimidated as many of the Amber Hill students did.

When visitors walked into the classrooms at Phoenix Park, which was a common occurrence, the students were unconcerned whether they were school inspectors, visiting dignitaries, or parents. They would always chat to adults, run around, misbehave, or swear at each other in the same relaxed manner regardless of who was with them. When the principal walked into lessons, the students would not change their behavior in any way, and those who were not doing work would continue not to do work. In many of my conversations with students and observations of them around the school, I was often reminded of the students at Summerhill school, the famously progressive English school described in Neill's (the principal) book: *Summerhill* (1985). Neill attributed the confidence and ease with which his students treated adults to the progressive approach of Summerhill school, which, he claimed, took away their fear and oppression (Neill, 1985).

The independence and responsibility encouraged in the students at Phoenix Park seemed to have a direct effect on their approach to mathematics. In a general sense, the students seemed less oppressed and constrained than many students of mathematics, and they seemed to take a more creative approach to mathematics than was typical for school students. In a questionnaire given to the students in Year 10, 82% of Phoenix Park students agreed with the statement "It is important in maths to use your imagination," compared with 65% of Amber Hill students—a statistically significant difference. The students' creative approach to mathematics was also demonstrated by an applied activity I gave them in Year 9 called *Planning a Flat* (apartment). In this activity, the students were asked to design a flat in a given space and locate and draw the rooms and furniture. A major, but unexpected difference between the students at Amber Hill and Phoenix Park related to the designs students produced. The students were invited to design a flat to suit a person or people of their choice (e.g., a student, a couple, a family, or themselves). The choice of rooms they would have in the flat was left entirely up to the students. All the students in both schools included in their designs at least one bedroom, bathroom, living room, and kitchen. However, approximately one third of the Phoenix Park students also included more unusual rooms. In the 89 designs produced by the students at Phoenix Park, there were 35 examples of *unusual* rooms including 7 games rooms, 4 soccer rooms (generally including small 5-a-side pitches), 3 indoor swimming pools, 3 studies, 2 hi-fi rooms, 2 children's playrooms, 2 cocktail bars, and 1 each of: a bouncy castle room, a pool room, a Jacuzzi, a computer room, a gym, a garage, a bowling alley, a utility room, a piano room, and a disco room. At Amber Hill, there were 99 flat designs that included 2 pool rooms, 2 swimming pools, 1 playroom, and 1 store room. It appeared that the students at Phoenix Park included the rooms they wanted to have in their flats, whereas the Amber Hill students included the rooms they thought they *should* have—the rooms of which they felt a teacher or I would approve.

The lack of constraint the Phoenix Park students experienced in these different domains, and the lack of domination or control that was imposed by teachers, seemed to have contributed toward the confidence of the students at Phoenix Park, the creativity they demonstrated, and the relaxed way in which they appeared to make and take decisions:

> A: That's the way I am . . . I just kind of do things in my own way, if it pulls off, it pulls off, if it doesn't then that's down to me. (Andy, PP, Year 10, RT)

SUMMARY

The mathematics approach at Phoenix Park was unusual particularly because of its openness, the degree of choice the students were given, the independence students were encouraged to develop, and the freedom the students had over their work environment and work rate. These features of the mathematics approach should be located within the overall context of Phoenix Park School, which was an unusually progressive institution that aimed to develop students' independence and decision-making abilities. The Phoenix Park approach was mathematically different from the majority of schools because learning mathematics was not based around the learning of different mathematical procedures. Rather, the students were engaged in activities and projects in which the need for certain mathematical methods became apparent. This approach necessitated a relaxation of the control teachers had over the structure and order of the classroom. The Phoenix Park teachers were not concerned about this, in line with their general approach to mathematics teaching and learning. Their concern was to give students mathematically rich experiences and help them use mathematics, rather than maintain order and a high work rate. They were concerned with the quality rather than the quantity of the students' mathematical experiences and with understanding rather than coverage. This meant that the Phoenix Park classrooms looked different from those of Amber Hill, and the students' experiences were also markedly different.

An important feature of Phoenix Park's approach was the care and attention the teachers paid to teaching students how to learn in an open system. Teachers had high expectations for their students, and they regarded the gap between where students were and where they needed to be (Black & Wiliam, 1998) as their teaching challenge. This approach required a lot from the Phoenix Park teachers. They needed to know where each of the projects may lead and which would be particularly interesting or important mathematical directions for students. They needed to know a lot about the students—what they knew and what would be most helpful for them to work on. They also needed to know the mathematics of the projects in some depth. Shulman and others have termed the combination of the knowledge of mathematics, students, and teaching ideas that the Phoenix Park teachers enacted *pedagogical content knowledge* (Ball & Bass, 2000; Shulman, 1986). The teachers used this knowledge in the pursuit of some important teaching practices. For example, they asked insightful questions that took students to the heart of the mathematical issues, they listened carefully to students' responses considering what they told them about the students' thinking, and they based their questions and guidance on what they learned in these interactions. As Ball and Bass have pointed out, such interactions require "pedagogically useful" knowledge (Ball & Bass, 2000, p. 89). It also seems important to note that the Phoenix Park teachers were extremely skillful in the ways in which they navigated students around the mathematical terrain. When students were stuck, teachers asked them to explain what they knew so far, they listened to students carefully, and they selected appropriate questions and interventions that helped students move forward. As a result of such careful teaching, many of the Phoenix Park students regarded mathematics to be a dynamic, flexible subject that involved exploration and thought. They valued the importance of mathematical processes, and the views they developed were, according to a wide range of literature (Doyle, 1983; Erlwanger, 1975; Schoenfeld, 1988), extremely unusual. Additionally, the students displayed a freedom, creativity, and lack of constraint in their interactions and behaviors, which appeared to derive directly from the approach of the school.

At Phoenix Park, the students did not believe lessons to be uniform and monotonous. Instead they regarded their lessons as varied, and their enjoyment of lessons depended on the particular activities they encountered. The students also displayed varied levels of engagement, which differed between students as well as between lessons and parts of lessons. A small but important proportion of the year group at Phoenix Park misbehaved in lessons and said they did not like the school's approach. However, it was difficult to know whether the students' lack of motivation caused their negative views about mathematics, whether it was the other way around, or whether neither one caused the other.

In the next chapter, I present the results of different assessments and consider the ways in which the difference between the two schools' approaches affected the students' understanding of mathematics.

6

Finding Out What They Could Do

There were fundamental differences between the learning experiences of students at Amber Hill, and Phoenix Park schools. At Amber Hill, the students were disciplined and hard working, and the mathematics they encountered was presented to them via a traditional, class taught, transmission model of teaching. At Phoenix Park, the students spent less time on task, and they only learned about new mathematical methods and procedures when they needed to use them in their projects. In assessments of their mathematical knowledge and understanding, broad differences would therefore be expected between the two sets of students. To investigate whether differences existed in the extent, nature, or form of students' understanding, I chose to use a variety of assessments: These included applied assessments, long-term learning tests, short contextualized questions, and the national GCSE mathematics examination. The different assessments involved are summarized in Table 6.1.

APPLIED ASSESSMENTS

One of the aims of my study was to investigate the notion of *situated learning* (Greeno & MMAP, 1998; Lave, 1988) – in particular, the ways in which students interacted with mathematics when it was encountered in different forms and settings. I knew that it would not be feasible to follow the students into mathematical situations in their everyday lives to do this, so I decided to give the students various applied activities within school. I then compared the students' responses to these activities with their re-

TABLE 6.1
Overview of Mathematics Assessments

Timing	*Form of Assessment*	*Students Involved*	*Research Aim*
Start of Y8	Seven contextualized, short questions	All case study cohort in both schools (n = 305)	To provide information on mathematical knowledge, use of mathematics in different contexts, and a baseline measure of the students' performance at the start of the research period.
End Y8	Architectural activity and related tests	Half of four groups in each school (n = 104)	To provide information on the students' use of mathematics in an applied activity, and their use of the same mathematics in a short test.
Mid-Y9	Long-term learning tests	Two groups in each school (n = 61)	To assess the students' knowledge of mathematics before it was taught, immediately afterward, and 6 months later.
End Y9	Nine contextualized short questions	All year group in both schools (n = 268)	To provide information on mathematical knowledge, use of mathematics in different contexts, and changes in performance between Y8 and Y9.
End Y9	Flat design and related tests	Four groups in each school (n = 188)	To provide information on the students' use of mathematics in an applied activity, and their use of the same mathematics in a short test.
End Y10	Analysis of GCSE answers	All GCSE entrants in each school (n = 290)	Knowledge of mathematics, analysis of use of mathematics in conceptual/procedural questions.

sponses to short, traditional tests that targeted the same areas of mathematics. The ways in which students react to applied tasks in school can never be used to predict the exact ways in which students will react to real-life mathematical situations. However, I believe that the degree of realism provided by applied tasks provides important insight into the different factors that influence a student's use of mathematics knowledge.

The Architectural Activity

In the summer of Year 8, approximately half of the students in the top four sets at Amber Hill (n = 53) and four of the mixed-ability groups at Phoenix Park (n = 51) were asked to consider a model and plan of a proposed house and to solve two problems related to district design rules. Students were given a scale plan, which showed different cross-sections of a house, and a scale model of the same house. To solve the problems, students needed to find information from different sources, choose their own methods, plan routes though the task, combine different areas of mathematics content, and communicate information. Because the students at Amber Hill were taken from the top half of the school's attainment range and the students at Phoenix Park were not, there was a disparity in the attainment levels of the samples of students. The Amber Hill student sample scored significantly higher grades on their mathematics NFER entry tests. However, my main aim was not to compare the overall performance of the students in the two schools, but rather to compare each individual's performance on the applied activity with his or her performance on a short written test. Approximately 2 weeks prior to the architectural task, the students took a pencil-and-paper test that assessed the areas of mathematics content I anticipated they would need to use in the activity.

The architectural activity (Fig. 6.1) comprised two main sections. In the first section, the students needed to decide whether the proposed house satisfied a council rule about proportion, which stated that the volume of the roof must not exceed 70% of the volume of the main body of the house. The students therefore needed to find the volumes of the roof and the house and find the proportion, of the roof volume to the house volume. To do this, students could use either the scale plan or model. The second council rule stated that roofs must not have an angle of less than 70 degrees. Therefore, the students had to estimate the angle at the top of the roof, which was actually 45 degrees, from either the plan or model. This was a shorter and potentially easier task.

In designing the activity, I formed questions that would require students to combine and use different areas of mathematics together. The in-

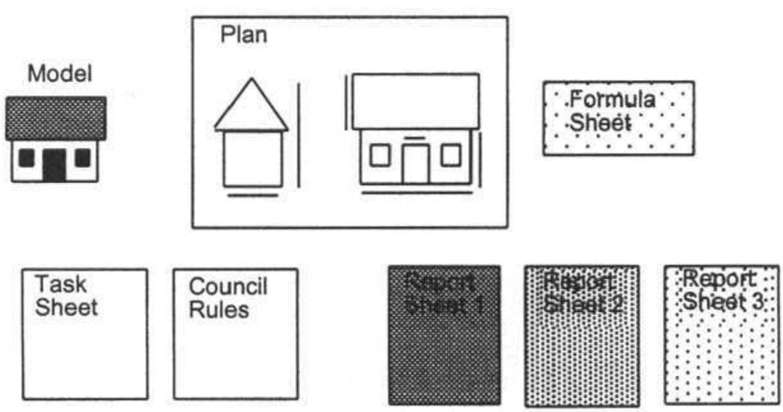


FIG. 6.1. The architectural activity.

dividual areas of mathematics involved (multiplication, division, area, volume, percentages, angles, measurement) were not particularly difficult, and they had been encountered by all of the students in all classes, but the demand of the task related to the need to choose, combine, and use different mathematical methods. Students had access to calculators at all stages of the activity.

Grades for the two sections were awarded as follows: A grade of 1 was given if the answer was correct or nearly correct, with one or two small errors; a grade of 2 was awarded if most or all of the answer was incorrect. All of the students made some attempt at the problems. In the test, the students were given three questions that assessed the mathematics involved in the architectural problem; they were asked to find the volume of a cuboid, the volume of a triangular prism (similar to the roof), and to calculate a percentage. Table 6.2 shows grades for the two aspects of the applied problem for students who answered the relevant test questions correctly. Table 6.3 shows grades for the two aspects of the applied problem for students who answered the relevant test questions incorrectly.

Tables 6.2 and 6.3 show that, in the roof volume problem, 29 Amber Hill students (55%) attained the highest grade of 1, compared with 38 (75%) of the Phoenix Park students, despite the fact that the Amber Hill students were taken from the top half of the school's ability range. Table 6.2 also shows that at Amber Hill 15 students (28%) could use the mathematics to answer a test question, but could not use it in the activity. This

TABLE 6.2
Problem Results for Students Who Answered the Relevant Test Questions Correctly

Grade	*Volume Amber Hill*	*Phoenix Park*	*Grade*	*Angle Amber Hill*	*Phoenix Park*
1	23	23	1	31	40
2	15	8	2	19	8
Total	38	31		50	48

TABLE 6.3
Problem Results for Students Who Answered the Relevant Test Questions Incorrectly

Grade	*Volume Amber Hill*	*Phoenix Park*	*Grade*	*Angle Amber Hill*	*Phoenix Park*
1	6	15	1	3	2
2	9	5	2	0	1
Total	15	20		3	3

compared with eight students at Phoenix Park (16%). Table 6.3 also shows that 15 students (29%) at Phoenix Park attained a 1 on the activity despite getting one or more of the relevant test questions wrong, compared with 6 students (11%) at Amber Hill.

In the test on angle, the students were given a 45-degree angle (the same angle as the roof in the activity) and asked whether it was 20 degrees, 45 degrees, 90 degrees, or 120 degrees. Fifty Amber Hill students estimated this angle correctly in the test, but only 31 of these students estimated the 45-degree angle correctly in the applied activity. At Phoenix Park, 40 of 48 students who recognized the angle in the test solved the angle problem. Paradoxically, the least successful students at Amber Hill were in Set 1, the *highest* group. Ten of the 14 students in the top set did not solve the roof volume problem, and 9 of the 14 students did not solve the angle problem. In both of these problems, this failure emanated from an inappropriate choice of method. For example, in the angle problem, the 10 unsuccessful students attempted to use trigonometry to decide whether the angle of the roof, which was 45 degrees, was more or less than 70 degrees, but they failed to use the methods correctly. Successful students estimated the angle using their knowledge of the size of 90-degree angles. Unfortunately, the sight of the word *angle* seemed to prompt many of the Amber Hill Set 1 students to think that trigonometry was required, even though this was inappropriate in the context of the activity. The students seemed to take the word *angle* as a cue to the method to use. Some of the students gained nonsensical answers from their misuse of trigonometry, such as 200 degrees, but they did not seem to realize that the 45-degree angle of the roof could not possibly have been 200 degrees.

The students undertook the architectural activity and associated tests at the end of Year 8—1 year after the start of their different approaches. At this stage, the difference between the mathematical behaviors of the two sets of students appeared to be emerging. This was particularly evident among students in Sets 1 and 2 at Amber Hill, who were less successful in the activity than students in Sets 3 and 4. At Phoenix Park, the students were slightly less successful on the test questions, which would be expected because the students were taken from a significantly lower attainment range, but the students were markedly more successful in the activities. The main problem that seemed to be experienced by the Amber Hill students related to an inability to decide what to do when they were not given explicit instructions. The students had learned appropriate mathematical methods, but when they were left to choose the methods to use, they became confused. For example, the students in Set 1 appeared to use trigonometry, rather than estimation, because the activity was about angles and they related angles to trigonometry. They were not able to see the redundancy of trigonometric methods within the situation.

Planning a Flat Activity

One year later, at the end of Year 9, all the students in the top four sets at Amber Hill (n = 99) and all the students in four mixed-ability classes at Phoenix Park (n = 89) were given a second applied activity and set of related tests. *Planning a Flat* (apartment) was adapted from a Graded Assessment in Mathematics (GAIM) activity of the same name (GAIM, 1988). Students worked on the activity and accompanying questions over the period of two consecutive lessons, each lesson lasting 1 hour. The activity and questions were given to complete classes. Again, the students at Amber Hill were of a significantly higher attainment range than the students at Phoenix Park measured on NFER tests.

In the first lesson, students were given a large-scale plan of an empty basement flat. The plan showed only the structural features of the flat—the external walls, windows, chimney breasts, and front door. The students were asked to decide on the intended owners of the flat and then appropriate rooms to put into the flat. Students then needed to draw rooms, doors, and furniture onto the plan using their knowledge of measurement and scale. On the flat's plan, students were given two important pieces of information. First, the scale of the flat was provided twice in two different forms. A 2-centimeter line showed the size of a meter at the bottom of the flat plan, and a box of information also gave the scale as 1:50 at the side of the plan. The second important piece of information concerned building regulations. A box at the bottom of the plan gave two regulations: (a) Each "habitable" room (i.e., living room, bedroom) must have a window in it, and (b) there must be two doors between a bathroom and a kitchen. Students were allowed to work together on the design of their flats if they wanted to, but they had to produce one design each.

In the second lesson, students were given two questions to answer, which related to their flats. The first question appeared in the students' instructions as follows:

> Carpet costs about £7.99 per square meter.
>
> a) *Roughly* how much would it cost to carpet all of the flat?
>
> Show all your working out.

The second question is reproduced in Fig. 6.2. Street doors must open to an angle of at least 115°. Will the street door of the flat pass this regulation? (The door is shown on the diagram below.)

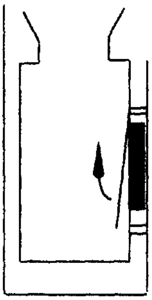


FIG. 6.2. Planning a flat question.

You *must not* use an angle indicator—explain how you have worked out your answer.

Approximately 1 month before taking the activity and related questions, the students were given a short written test that assessed all of the mathematics featured in the activity and related questions. In the short written test, the students from Amber Hill attained significantly higher grades on questions assessing area, angle, and percentage than the students at Phoenix Park; there were no significant differences on the question assessing scale. These differences were mainly because of the high success rate of the Set 1 students at Amber Hill; when these students were taken out of the sample, the only significant difference between the two schools occurred in the question on area. Despite the Amber Hill students' apparent competence with the mathematical procedures involved, the Phoenix Park students attained significantly higher grades on all sections of the applied activity and related questions.

The students' flat designs were assessed using the GAIM (1988) criteria for the activity. Grade 1 is the highest grade, grade 5 the lowest. High grades are given if students make correct measurements, use scale appropriately, take account of the building regulations, and produce well-proportioned designs. The two groups of students' produced the results shown in Table 6.4.

These results show that there were vast differences between the performance of the students at the two schools, with Phoenix Park students gaining significantly higher grades, despite the fact that the students were taken from a significantly lower attainment range. The main difference be-

TABLE 6.4
Flat Design Results (%)

%	*1*	*2*	*3*	*4*	*5*	*n*
AH	31	24	7	18	19	99
PP	61	6	8	13	12	89

tween the two schools was that 61% of Phoenix Park students produced well-planned designs, with appropriately sized and scaled rooms and furniture, compared with only 31% of Amber Hill students. Twenty-four percent of the Amber Hill students drew rooms of an inappropriate size or that contained wrongly scaled furniture and doors, compared with 6% of the Phoenix Park students. This was despite the fact that 90% of Amber Hill students successfully used scale in the short written test.

Another major difference between the two schools was described in chapter 5. This related to the types of rooms the students designed. As noted earlier, 33% of the Phoenix Park students included unusual rooms such as disco rooms and bowling alleys in their flats, compared with approximately 3% of Amber Hill students. In general, the inclusion of these more unusual rooms at Phoenix Park did not mean that the students produced unrealistic designs with inappropriately sized rooms. Many of the designs were ingenious, entailing a creative use of space with interlocking rooms that saved on redundant hall or corridor space. In effect, the students often gave themselves a more demanding cognitive task, but managed to attend to the given rules and the constraints of size and scale to produce impressive designs. This reflected a general and quite marked difference between the two schools. Many of the Amber Hill designs were inaccurate, sketchy, and basic despite the obvious commitment and enthusiasm shown by the students during the activity. The Phoenix Park designs were of a much higher standard, and they included carefully and accurately constructed designs and furnishings. Students at both schools reported enjoying the activity immensely, particularly the Amber Hill students, many of whom asked if they could do more work of a similar nature.

In the second lesson, the students were asked to answer the three questions that related to their flat designs. The first question asked the students how much it would cost, "roughly," to carpet all of their flats using carpet of a given price. Students attained grade 1 if they gave an appropriate estimation, grade 2 if they worked out the exact answer, and grades 3 to 7 depending on the number of mistakes they made (see Table 6.5).

In the question on area, there were significant differences between the two schools, with 71% of Phoenix Park students attaining the highest grade, compared with 38% of Amber Hill students. Grade 1 was given for answers that gave a correct *approximation* of the cost of carpet, which was the requirement of the question. Grade 2 was given if the students calculated the exact area of the floor space of the flat, subtracting the space taken up by the fireplace and other protrusions. Thirty-four percent of Amber Hill students attained this grade. The decision to work with this degree of accuracy was not sensible in the context of the activity because

TABLE 6.5
Area Question Results (%)

%	1	2	3	4	5	6	7	*n*
AH	38	34	10	2	9	7	0	88
PP	71	17	1	1	0	6	4	72

carpet would need to be bought for the area, including the fireplace spaces. If these spaces were subtracted from the length of carpet bought, there would not be sufficient carpet for the flat. Quite apart from this constraint, the question asked the students to work out roughly how much the carpet would cost, and the word *roughly* was highlighted.

The response of the Amber Hill students was interesting because it demonstrated again the influence of certain goals or constraints on the way in which students responded to the question. The students who used an exact measurement of floor space to answer the question did not show a good understanding of the demands of the context, although they had worked on their designs for the entire previous lesson. This was probably because they were doing what they thought was expected of them, which meant working with the numbers and ignoring the situation or context in which they were placed.

In the final question, which asked whether the street door could open to an angle of at least 115°, grade 1 was given for correct answers and grades 2 to 4 for various incorrect answers. This question's results are shown in Table 6.6. These results show that the Amber Hill students were relatively unsuccessful at estimating an angle within the context of a problem despite the fact that 96% of Amber Hill students successfully estimated a similar angle in the short written test.

The results of this applied activity reveal that there were significant differences between the performance of the students in the two schools in their flat designs, their use of area, and their estimation of an angle. The lack of success among the Amber Hill students on various aspects of the activity was not caused by their lack of mathematics knowledge, but appeared to derive from the goals the students formed in relation to the activity. In producing their flat designs, the Amber Hill students did not seem to work with the freedom of the Phoenix Park students. The Phoenix

TABLE 6.6
Angle Question Results (%)

%	1	2	3	4	*n*
AH	43	16	39	2	89
PP	75	10	10	6	71

Park students produced more unusual and creative designs, which were also more accurate and appropriately sized and scaled. The Amber Hill students may have failed to make use of their knowledge of scale and measurement because they had not been told to demonstrate such knowledge in the activity. In the question on area, the Amber Hill students were able to work out the mathematical methods they should demonstrate, and many of the students gave answers that were "too" accurate for the situation or context. I interviewed 10 of these students after the activity and asked them why they had done this. They said that they used this degree of accuracy because they thought they had to "show their maths." The students demonstrated quite clearly the influence of nonmathematical goals on their choice of mathematics method. If the students had been asked why they attempted to use trigonometry, rather than estimate the angle in the architectural problem, they probably would have said the same thing—to show the methods they had learned. A further 28% of Amber Hill students were unable to work out an area of any accuracy, compared with 12% of Phoenix Park students. In the question on angle, many of the Amber Hill students again failed to show their knowledge of angle that they had shown in the test.

The Amber Hill students' performance on various aspects of the flat design task showed they had difficulty making use of the mathematics they learned in an applied situation. This did not appear to be due to a lack of mathematics knowledge, but the ways in which the students *interpreted* the demands of the activity. This is considered in more depth in the next chapter. At Phoenix Park, the students performed well on all aspects of the task and related questions despite the fact that the students' ability range was lower than that of the Amber Hill students.

LONG-TERM LEARNING TESTS

In this assessment, students were tested on a piece of their school work immediately before being taught the work (pretest), immediately after completing the work (posttest), and then 6 months later (delayed posttest). On each of the three assessment occasions, the students took exactly the same test. The tests were designed to assess the learning that took place on a particular topic, in a similar style and format to the actual work. Because the Amber Hill students were taught in sets, their work was usually targeted at specific levels of content. This made the design of the assessment questions straightforward. I essentially designed questions that were replicas of the questions they had worked on in their SMP textbooks, with different numbers and contexts. In Phoenix Park, the design of the assessment questions was extremely difficult because the students were working at different levels of mathematics. Therefore, the results for the Amber Hill students are more valid than those for the Phoenix Park students. For this reason, I do not expand on this exercise in detail, but I have provided a summary of the results because, even with these limitations, they were interesting.

At Amber Hill, the two groups assessed were a Year 8 top set and a Year 9 set 4, both taught by Edward Losely. Both groups were taught using the typical Amber Hill approach, with the teacher explaining methods on the board, followed by the students practicing the methods in exercises. The top set group was taught at a relatively fast pace as was normal for the school. Both groups worked for about 3 weeks on the topics assessed.

The experiences of the two groups at Phoenix Park differed in more fundamental ways. One group was a Year 8 that was working on what Martin described as "the most didactic piece of teaching" they ever did at the school. This consisted of Rosie teaching the students how to do long division without a calculator on the board and then letting them explore division patterns. The work lasted for only 2 lessons. The Year 9 work, also taught by Rosie, was a more typical Phoenix Park project on statistics that lasted for about 3 weeks. A summary of the main results follows.

- The least successful group of the four was the top set Year 8 group at Amber Hill. This group learned and remembered 9% of the work they had been introduced to over a 6-month period. The Year 9 group learned and remembered slightly more of their work—approximately 19%. This was despite the fact that the Amber Hill students were given tests that were exact replicas of their exercise book questions, with different numbers or contexts. At Phoenix Park, the Year 8 group learned and remembered only 16% of the work over a 6-month period; the most successful group was the Year 9 Phoenix Park group, who remembered 36% of the statistics they used during their open-ended projects.
- Because the Phoenix Park tests had to be pitched in the middle of the group and some students may not have worked on aspects of mathematics that were assessed, a better comparison of the two schools is provided by the ratio of the proportion of questions answered correctly in the posttest to the proportion answered correctly in the delayed posttest. These ratios are displayed in Table 6.7.

The results of the long-term learning tests show that the Phoenix Park Year 9 students were more successful than students at Amber Hill and the Year 8 students at Phoenix Park. The results of the Amber Hill tests are particularly interesting because the close match between the tests given to students and the work in their textbooks meant that the tests could give a

TABLE 6.7
Ratio of Success on Long-Term Learning Questions, Posttest: Delayed Posttest

	Posttest: Delayed Posttest	
School	*Year 8*	*Year 9*
Amber Hill	3:1	2:1
Phoenix Park	3:2	6:5

relatively realistic picture of the students' learning. These tests show that the Amber Hill students could remember a reasonable proportion of their work immediately after their lessons, but 6 months later the top set Year 8 group had forgotten two thirds of what they had learned and the Year 9 Set 4 group had forgotten half of what they had learned. At Phoenix Park, the Year 8 students had forgotten one third of their work and the Year 9 students had forgotten one sixth.

MORE TRADITIONAL MATHEMATICS ASSESSMENTS

Introduction

The superiority of the students' performance at Phoenix Park in applied mathematics assessments is probably not surprising given the students' greater experience of open-ended mathematical activities in lessons. At Amber Hill, the students spent the vast majority of their time working through short, closed exercises assessing rules and procedures that they had been taught by their teachers. Part of the reason that the school chose to teach in that way was to provide the students with a good preparation for examinations that assess mathematics in a similar format. This section presents the results of two different assessments that gave the Amber Hill students the opportunity to use the mathematics they had learned in a more familiar format.

Year 9 Context Questions

At the end of Year 10, the students were given the same short questions set in different contexts that were given to them at the beginning of the research study. These questions assessed conservation of number, number groups, fractions, perimeter, and area. On the five questions that assessed fractions and conservation of number, there were no significant differences between the schools. On the two number group questions, the Amber Hill students attained higher grades mainly because a large proportion of Phoenix Park students did not answer these questions. On the two questions involving perimeter and area, the Phoenix Park students attained higher grades. Taken overall, the performance of the two sets of students on these tests was therefore broadly comparable—this result, which was in some senses surprising, is illuminated further by a consideration of the students' performance on GCSE examinations.

Year 10 GCSE Examinations

At the end of Year 10, students take national examinations in each of their subjects—normally around eight subjects. If a teacher thinks a student has no hope of attaining any grade, he or she may not enter the student for the examination, but this is unusual and there is an expectation that all students, except those with severe special educational needs, will take a GCSE examination in each of their subjects. All subjects, including art, music, and Physical Education, are examined. At Phoenix Park, the teachers explicitly stated that their approach was intended to give students a comprehensive mathematical understanding that would help them throughout life and was not narrowly targeted at examination success:

> Its approach is to try and encourage students to be able to use their mathematical knowledge, as much as to acquire new bits of mathematical knowledge so, the two go hand in hand and we try and, I suppose we focus, have focused in the past, to a great degree on the process of using mathematics rather than on acquiring the bits of archaic knowledge that people are required to do to get through the hoops of GCSE. So, it's very much of an approach where we want the students to acquire skills, that they are going to be able to use and apply in the rest of their lives, rather than to get some kind of body of knowledge. (Martin Collins, mathematics coordinator)

However, success on the examination was important for the students and the school. It was surprising then that the teachers at Phoenix Park were somewhat cavalier about examination preparation, and there were a number of ways students were disadvantaged when they came to take the examination. One important disadvantage that students faced was not being provided with a calculator on examination day, as is normal practice in schools. This was important for the Phoenix Park students because many of them could not afford to buy their own and they expected calculators to be provided:

L: Like the day before they told us all the equipment we needed and we had to go out and buy it and if you didn't have any money then you didn't have the equipment.
H: Like it was your responsibility to take a calculator in and that.
L: Yeah, like they usually supply them in lessons, then they didn't in the exam. (Helen & Linda, PP, Year 10, MC)

The fact that the school did not lend the students calculators for the examination was fairly indicative of the school's relaxed approach to examinations in general. Martin said they could not supply calculators because the mathematics department did not have the money to buy them: They had bought enough calculators at the start of the year, but they could not replace those that were lost or stolen.

The students may also have been disadvantaged by the relaxed atmosphere of the school, which meant that few of the Phoenix Park students were "geared up" for their GCSE examinations. Indeed, many of the Phoenix Park students reported that they had not bothered to "revise" (review) for the examination:

H: I can't say anyone I know is bothered about their GCSEs, I don't think we're revising or bottling down or anything, I think it hasn't hit us yet.
L: Yeah, I haven't done anything yet.
H: No, me neither.
L: Now I don't think I've got any time left to revise what's going to be in the exam and then you just leave it 'cause you don't know enough. (Helen & Linda, PP, Year 10, MC)

At Phoenix Park, students worked on open-ended problems throughout most of Years 8, 9, and 10 in mixed-ability groups, apart from the few months leading up to the examination, when the teachers grouped the students into one of three examination groups—foundation, intermediate, or higher. At this time, they taught them more standardized methods and procedures that they thought they would need in the examination. Part of the reason they needed to do that was because students had generally only encountered methods when they needed to use them in their projects, so the methods students knew varied. The last few months at Phoenix Park were spent tying up loose ends and introducing methods that had been met unevenly or not at all. At this time, the teaching and learning environments at Phoenix Park appeared similar to those at Amber Hill, with students watching teachers work through mathematics methods on the board and working through past examination papers. Despite this similarity, the students experienced a major difference at the two schools: At Phoenix Park, the teachers rarely mentioned the examination; at Amber Hill, the teachers constantly stressed its importance and reminded students of the need to prepare for it at regular intervals. Indeed, the Amber Hill teachers did not make any pretense of preparing students for more open, applied, or realistic assessments of their knowledge. They were clear that their job was to prepare students for the GCSE examination in the best way possible. The Amber Hill students were also convinced of the aim of mathematics lessons, and they reported that the high degree of motivation and hard work they demonstrated in lessons derived from their desire for GCSE success:

JB: So if you all dislike it so much, why do you work so hard in lessons?

C: Because we want to do well, maths GCSE is really important, everyone knows that. (Chris, Amber Hill, Year 10, Set 4)

The pressure the students received to do well at Amber Hill may have disadvantaged them in the examination in the same way that the lack of pressure to do well may have diminished the Phoenix Park students' capabilities. However, there were a number of indications that the Phoenix Park students faced a range of real and important disadvantages when they took their GCSE examinations, with which the Amber Hill students did not have to contend. These factors made the results from the two schools somewhat surprising.

At the end of Year 10, Amber Hill entered 182 of the 217 students in the grade level for GCSE mathematics; this amounted to 84% of the cohort. At Phoenix Park, 108 of the 115 students in the grade level were entered for the examination, which was 94% of the cohort (schools do not enter the students if they think they will definitely fail the examination). The two schools used different examination boards, but Tables 6.8 and 6.9 give the results of the students at each school, as well as the national results for the different examination boards. A* is the highest grade possible. Table 6.10 shows the A*–C and A*–G results from both of the schools. Any grade from A* to G is a pass. Grades U, X, and Y are fail grades.

The GCSE results at the two schools show that similar proportions of students at the two schools attained A*–C grades, but significantly more Phoenix Park students attained A*–G grades. This was despite the similarity in the cohorts at the end of Year 8, the increased motivation of the Amber Hill students to do well, the examination-oriented approach at Amber Hill, and the lack of calculators at Phoenix Park. Indeed, six Phoenix Park students wrote onto their actual GCSE papers, "I haven't got a calculator,"

TABLE 6.8
Amber Hill GCSE Results

	A*	A	B	C	D	E	F	G	U	X	Y	n
Students (n)	0	1	4	20	25	40	37	26	19	10	0	182
% entry	0	0.5	2.2	10.9	13.7	22.0	20.3	14.3	10.4	5.5	0	182
% cohort	0	0.5	1.8	9.2	11.5	18.4	17.1	11.9	8.8	4.6	0	217
National average	3.2	8.3	16.9	27.2	13.3	14.2	10.5	4.4	2.0	NA		100

TABLE 6.9
Phoenix Park GCSE Results

	A*	A	B	C	D	E	F	G	U	X	Y	n
Students (n)	1	2	1	9	13	28	27	20	5	1	2	108
% entry	1	1.9	1	8.3	12	25.9	25	18.5	4.6	0.93	1.9	108
% cohort	0.9	1.7	0.9	7.8	11.3	24.3	23.5	17.4	4.3	0.87	1.7	115
National average	0.2	2.0	7.3	15.1	16.8	18.4	16.5	16.2	7.5	NA		100

TABLE 6.10
Comparison of GCSE Results (%)

	Entry		*Cohort*	
	AH	*PP*	*AH*	*PP*
A*–C	13.7	12.0	11.5	11.3
A*–G	84.1	93.5	70.5	87.8
% entered	84	94		

and at frequent points in the examination they wrote out the method they had used in the questions, but did not evaluate the answers, thereby losing marks. Despite these different factors, significantly more of the Phoenix Park students passed the GCSE examination than Amber Hill students. The A*–C results for both schools were lower than national averages, but this would be expected from the intakes of the two schools. What would not have been expected was the similarity in A*–C grades at the two schools, the increased proportion of A*/A grades at Phoenix Park (2.9% of the Phoenix Park students, compared with 0.5% of the Amber Hill students), and the proportion of A*–G passes at Phoenix Park, which was higher than the national average for the examination. However, I was not unduly surprised by the two sets of results or the Phoenix Park students' superior performance. This was not because the students at Phoenix Park knew more mathematics—they did not—but because they had developed different *forms* of knowledge and understanding.

I was given special permission to visit the examination offices where completed papers are held under lock and key. There I recorded the marks that each student attained for every question on the GCSE examination papers. I had previously divided all of the questions into the categories *procedural* and *conceptual*. Procedural questions were those questions that could be answered by a simplistic rehearsal of a rule, method, or formula. They were questions that did not require a great deal of thought if the correct rule or method had been learned. An example of such a question would be, "Calculate the mean of a set of numbers," provided, of course, that students had learned how to calculate a mean, students did not have to decide on a method to use, nor did they have to adapt the method to fit the demands of the particular situation. An example of a conceptual question was, "A shape is made up of 4 rectangles, it has an area of 220cm^2. Write, in terms of x, the area of one of the rectangles" (diagram given). Such a question requires the use of some thought, and rules or methods committed to memory in lessons would not be of great help in this type of question. My rule in allocating questions was therefore: If the question could be answered mainly from memory, it was procedural; if it also or instead required a substantial amount of thought, it was conceptual. All the examination papers from both examination boards included both procedural and conceptual questions, with approximately twice as many procedural as conceptual questions. An analysis of the procedural and conceptual questions that students answered correctly and incorrectly in each school reveals a significant difference between the schools. The box and whisker plots given in Figs. 6.3 and 6.4 show the distribution of the percentages of students attaining correct answers for the two different types of question at each school.

The conceptual questions were often, by their nature, more difficult than the procedural questions—even for a student who had both learned and understood mathematical rules and procedures. Therefore, the students at both schools would be expected to answer more of the procedural questions correctly. At Amber Hill, there was a significant difference between the percentages of students answering procedural and conceptual questions correctly. At Phoenix Park, the percentages of students correctly answering the conceptual questions was, on average, only slightly lower than the percentages solving the procedural questions.

Students at both schools were entered for one of three levels of GCSE paper—higher, intermediate, or foundation. At Phoenix Park, the students who answered the greatest proportion of conceptual, as opposed to procedural, questions were those who took the higher level examination (same number of each type), followed by the intermediate level students (9 conceptual questions for every 10 procedural questions), followed by the foundation level students (6 conceptual questions for every 10 proce-

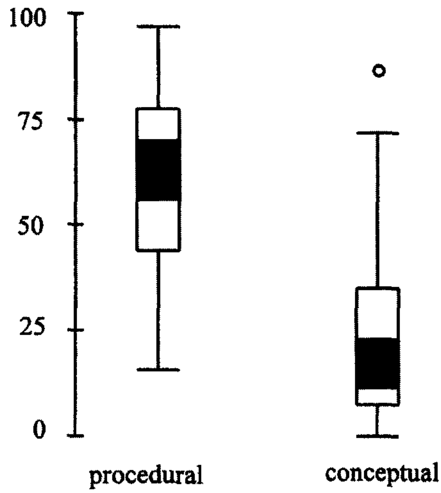


FIG. 6.3. Distribution of student scores for procedural and conceptual questions at Amber Hill.

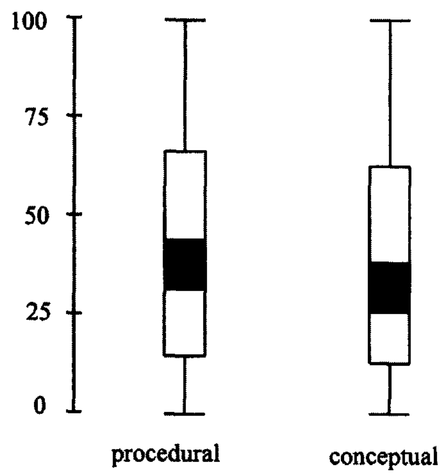


FIG. 6.4. Distribution of student scores for procedural and conceptual questions at Phoenix Park.

dural questions). This trend across the three levels of paper may simply reflect the nature of mathematical confidence and ability, with the more competent students, entered for the higher papers, being more willing and able to tackle questions with a conceptual demand. However, at Amber Hill, it was the *intermediate* level students who answered the highest proportion of conceptual questions (8 conceptual questions for every 10 procedural), followed by the foundation and higher level students (5 conceptual questions for every 10 procedural questions). Thus, the higher level Amber Hill students went against the trend displayed by the other students. At Amber Hill, all the higher level students were taught in the top set, and it seems likely that the speed at which they encountered work and the closed and rule-bound nature of their experience may have inhibited their performance on these questions. Chapters 9 and 10 explore the possibility that the top set students at Amber Hill were disadvantaged by their placement within this set.

Analysis of the different questions answered correctly by students taking the different levels of paper also reveals that, although similar proportions of students at both schools attained grades A*–C, these grades were achieved in different ways. The higher level students at Amber Hill correctly answered two procedural questions for every one conceptual question, whereas the higher level Phoenix Park students attained equal numbers of each question correct. The main source of disadvantage for the potential A*–C grade students at Amber Hill seemed therefore to be the conceptual questions, which took up approximately one third of the examination paper. At Phoenix Park, the students attained equal proportions of each question correct, although many of the conceptual questions were quite demanding. This suggests that the students would have done much better at this level if they had been taught more of the procedures assessed in the examination. The responses of the students who could have attained A*–C grades also indicated that their lack of knowledge of formal procedures fazed them in the examination:

> L: There were loads we hadn't done weren't there? There were all those ones with weird equations that we'd never seen. (Lindsey, PP, Year 10, JC)

The main source of disadvantage for the potential A*–C students at Phoenix Park seemed to be their lack of knowledge of procedures, which was important because the procedural questions took up two thirds of the examination paper. Despite this, the overall attainment of the two cohorts of students was broadly equivalent.

A consideration of the proportion of students attaining grades A*–G at each school shows that the Phoenix Park students were significantly more successful. At Amber Hill, only 84% of entrants and 71% of the cohort attained grades A*–G, compared with 94% of Phoenix Park entrants and 88% of their cohort. Indeed the A*–G results for Phoenix Park were better than the national average, although the achievement of the cohort was considerably lower than the national average on entry to Phoenix Park. In many ways, this was a remarkable achievement. The distribution of grades in each school shows that this difference seemed to be largely due to the fact that more of the Phoenix Park students attained grades E, F, and G (57% at Amber Hill, 69% at Phoenix Park), whereas more of the Amber Hill students failed the examination (16% at Amber Hill, 7% at Phoenix

Park). This was despite the fact that Amber Hill did not enter 35 (16%) of their students, and Phoenix Park entered all but 7 (6%) of their students.

These results give clear evidence of a superior performance from the Phoenix Park students, particularly those who were entered for the foundation level papers. An examination of the types of questions answered across the foundation level papers shows that students at both schools answered correctly approximately two procedural questions correctly for every one conceptual question. Thus, the Phoenix Park students did not attain higher grades because they answered more of a particular type of question correctly; rather they attained higher grades because they answered more of *both* of these types of question correctly. One source of disadvantage for the Amber Hill students was probably the fact that the students who should have attained grades E, F, or G were in low sets, whereas the Phoenix Park students were in mixed-ability groups. This is considered in more detail in chapter 10. The other likely source of disadvantage for the Amber Hill students was simply that they had developed a less effective mathematical understanding. This too is considered in more detail in the next chapter.

At Phoenix Park, there was a group of badly behaved and apparently unmotivated students in the year group. However, a comparison of NFER entry results and GCSE results show that these students did not underachieve on the GCSE examination in relation to other students. This could mean that the students engaged with their mathematics for at least some of the time and that their bad behavior (and other students' good behavior) was a less effective measure of their mathematics learning than would normally be assumed. A comparison of the three teachers at Phoenix Park also showed that the students in Jim's groups, who experienced the most freedom, performed as well or better than the students in the classes of Martin and Rosie, the stricter teachers, although all groups had a similar attainment range when they began at Phoenix Park.

An overall consideration of the GCSE results indicates that if Amber Hill and Phoenix Park's approaches were to be evaluated in terms of examination success alone, the Phoenix Park approach must be considered more successful. This is despite the fact that the Phoenix Park students were not used to examinations and their school's approach could not, in any sense, be regarded as examination-oriented. A consideration of the students' performance on procedural and conceptual questions on the GCSE examination shows that the students at the two schools attained broadly similar grades in different ways. The Amber Hill students were much more successful on the procedural questions, which suggests that their examination performance would be enhanced if they were able to think about and solve more of the conceptual questions. The development of this capability would probably also advantage students in many other situations because the conceptual questions required a depth of thought that would be useful in a number of applied and real-world settings. The Phoenix Park students would probably achieve greater examination success if they learned more of the standard mathematics procedures that are assessed in the GCSE examination, but it is unknown whether these would help the students in any other situation than a mathematics examination. This raises questions about the appropriateness of the mathematics assessed in the GCSE examination, an issue to which I return in the final chapter.

DISCUSSION AND CONCLUSION

The results of all the assessments reported in this chapter are broadly consistent. They show that many more of the Phoenix Park students had developed a mathematical understanding that they were able to make use of than the Amber Hill students. This was demonstrated in various applied situations and conceptual GCSE questions. Even within more traditional assessments, the Phoenix Park students performed as well or better than the students at Amber Hill. These results all seem to indicate the same phenomenon: The students at the two schools had developed a different *kind* of mathematics knowledge. The Phoenix Park students did not have a greater knowledge of mathematical facts, rules, and procedures, but were more able to make use of the knowledge they did have in different situations. The Phoenix Park students showed they were flexible and adaptable in their use of mathematics probably because they understood enough about the methods they were using to utilize them in different situations. The Amber Hill students developed a broad knowledge of mathematical facts, rules, and procedures that they demonstrated in their textbook questions, but they found it difficult to remember these methods over any length of time, and they did not know enough about the different methods to base decisions on when or how to use or adapt them. Further evidence of these important differences in the students' mathematical behavior is presented in the next chapter, which explores the apparent differences in the students' knowledge and understanding.

7

Exploring the Differences

In chapter 6, I presented the results of a number of different assessments. These assessments, taken together, indicate some important differences between the capabilities of the students at the two schools. In this chapter, I show that while the performance differences on these assessments were not always large, they reflected an important variation in the nature of the students' understanding. The students' reflections and ideas collected from interviews over the 3 years provided fascinating insights into their mathematical knowledge, beliefs, and understanding that I explore now.

AMBER HILL

Performance Patterns

There was evidence from both lesson observations and the assessments shown in the last chapter that the Amber Hill students were able to use the mathematics knowledge they had learned when the requirements of questions were explicit. This meant that they could work through their exercises in class with relative ease; they performed well on all of the short, written tests that accompanied the applied activities, and they were able to answer many of the procedural GCSE questions. The difficulties seemed to occur for the students when the requirements of questions were not explicit, when they needed to use some mathematics after a period of time, when they had to apply mathematics, and when they needed to combine different forms of mathematics. The mathematical competencies students displayed in different situations reflected both their understanding of mathematics and the beliefs that the students had developed *about* mathematics. These two aspects of mathematical capability worked together in interesting ways explored in this and the next chapter.

At Amber Hill, many of the mathematics lessons were rapidly paced, closed, and procedural. This seemed to have had a clear impact on the students, causing them to develop a shallow, procedural kind of knowledge and a perception that mathematics was all about learning and remembering rules and formulae. Neither the students' views nor the procedural nature of their learning were surprising given that the students spent much of their time in lessons reproducing methods, and they were not given the time nor the encouragement to think about them deeply. The students became concerned about their mathematical performance when they took their "mock" GCSE examinations. This is a fairly important rehearsal for the real examination when students take an examination paper from a previous year under the same conditions. Until that time, the students had thought that they would be successful in mathematics if they learned all the rules and formulae they were introduced to in their lessons. In the mock GCSE examination, the students found that this was not the case:

> A: It's stupid really 'cause when you're in the lesson, when you're doing work—even when it's hard—you get the odd one or two wrong, but most of them you get right and you think well when I go into the exam I'm gonna get most of them right, 'cause you get all your chapters right. But you don't. (Alan, AH, Year 10, Set 3)

The students encountered a variety of difficulties in both the mock and actual GCSE examinations. For example, they could not remember many of the procedures they had learned over time. This was demonstrated by the long-term learning assessments and supported by the students' comments in interviews:

> S: Usually, like I know that pi is equal to 3.14 because it's easy to remember but I don't actually remember like the diameter, how to find out the diameter of a circle 'cause we done that a few weeks ago.
>
> B: No I can't remember that, like the circumference and the radius.
>
> S: I wouldn't know now how to think about it, like we done that—what about 3 weeks ago? and I could do it when we finished it but I don't think I'd remember it now. (Sam & Bridget, Amber Hill, Year 9, Set 3)

Many of the Amber Hill students talked in similar terms about the difficulty they experienced using mathematics after a period of time. Sam's comment gives some indication of the reason for this because she said that she "wouldn't know now how to think about it," suggesting that because her memory of the procedure had gone she would not be able to think about the mathematics. This suggests that students were disadvantaged in two related ways. First, they experienced difficulties because of their belief that they had to rely on their memory to solve mathematical problems. Then they experienced difficulties because the ways in which they had learned methods had not given them access to a depth of understanding that helped them to remember methods. This meant that students had problems even when they were presented with straightforward questions that assessed isolated mathematical concepts in forms that were very familiar to them. For example, 93% of students who took the intermediate GCSE paper answered a question on simultaneous equations. All of these students attempted to use the standard procedure they had learned, but only 26% of students used the procedure correctly. The rest of the students used a confused and jumbled version of the procedure and received no marks for the question.

A second problem was experienced by students when they needed to use different types of mathematics within the same activity. For example, in the first part of the roof problem in the architecture activity, students needed to use measurement, scale, volume, and percentage. The combination of these methods in the same problem seemed to cause difficulties for the Amber Hill students, and the students reported similar difficulties in the GCSE examination:

M: 'Cause in the exam, we only had about 2 of them questions from class, in the whole exam – probably the whole year got them right.

JB: What sort of questions – when you say there were only 2 of them, what sort were they?

M: Like, if you have this and that number and then, how do you do it?

JB: So what was the rest of the exam if it wasn't that?

M: It was jumbled up, it was like ratio and then it was like digits and then the next question was that then it went back to ratios again, then it went to bearings, then it went to that and that, you see? (Marco, AH, Year 10, Set 4)

When the Amber Hill students worked through their textbooks, they learned one procedure and then practiced it; in the examination, they needed to think about and combine different procedures and flexibly switch between different procedures in different questions. This caused difficulties for many of the students.

A third and even bigger problem was created for the students in situations when they needed to apply the methods they had learned. This was clearly demonstrated by the two applied assessments reported in the last chapter. These showed that students successfully used knowledge in tests, but failed to make use of the same knowledge in more applied activities. In the GCSE examination, the students also reported that they were unable to apply the methods they had learned:

> L: Some bits I did recognize, but I didn't understand how to do them, I didn't know how to apply the methods properly. (Lola, AH, Year 10, Set 3)

Thus, even when students knew they had learned an appropriate method, they often could not do anything with it.

The difficulties the students experienced all seemed to relate to or fit within an overarching phenomenon, which concerned the way in which students interpreted the demands of situations. They knew appropriate mathematical methods to use, but they were rarely able to decide on appropriate situations in which to use them.

Interpreting the Demands of Situations

In the architecture problem described in the last chapter, a number of the Amber Hill students did not calculate the volume or angle of the roof correctly. This was not because they could not perform calculations with volume or angle, but because they needed to interpret the question in order to determine what to do. Many of the students were unsuccessful because they saw the word *angle* and thought they should use trigonometry; it was their interpretation of the demands of the situation that failed them. There were other indications that the Amber Hill students were unsuccessful in the two applied situations that they were given in Years 9 and 10 because these required them to interpret the activities and decide what to do. This confusion was similar to the confusion students experienced when they moved between different exercises in their textbooks. They could perform the mathematical procedures, but they could not work out which procedures were needed.

In the examination, this was also a major concern for students. They related many of the difficulties they faced to the fact that the examination questions did not contain any cues in the way that their textbook questions did. In the textbook questions, the students always knew what method to use—the one they had just been taught on the board. If a question required something different or additional to this, there was always some clue in the question that would indicate what they had to do. In the examination, the questions did not simply require a precise and simplistic rehearsal of a rule; they required them to understand the questions and know what the questions were asking them:

A: We had one question, didn't we, and it's got like, what was it?, something stupid like . . . it was symmetry, you know, lines of symmetry, we had to change it round and it was, oh, it just said like, I've forgotten what it said now, but it had like this sentence and you thought—what do I do? It didn't explain what you had to do in the paper and it was about 9 marks for that and you lost 9 marks just because it didn't tell you what to do. (Andy, AH, Year 10, Set 3)

G: Yeah in the exam it's like essays and that and . . . questionnaires. . . they're like misleading, and it's the same with graphs, they're misleading, graphs, and the questions, they're really misleading and if you can't understand one part you can't get the next part, and then you start panicking, but in the book and in the class it more or less explains itself. (Gary, AH, Year 10, Set 3)

In their textbook lessons, the students had not experienced these demands because the textbook questions always told them what to do; they always followed on from a demonstration of a principle, method, or rule. Unfortunately, the textbook questions never required students to decide on a method to use and, as Gary said, "in the book and in the class it more or less explains itself."

The students' inability to succeed on questions that did not indicate the correct procedures to use can be related back to the students' belief that mathematics was a rule-bound, memory-based subject. The students could not think about and decide what was required of them in the examination because they believed that thinking was not what they were meant to be doing.

S: Yeah you have to learn it so that you can tell the difference in the question as to which rules you use. (Sara, AH, Year 10, Set 3)

They had been trained to learn rules and spot clues in questions, rather than interpret situations mathematically:

L: In maths you have to remember, in other subjects you can think about it, but in exams the questions don't really give you clues on how to do them. (Lorna, AH, Year 10, Set 1)

In this extract, Lorna described quite clearly the problem she faced. She could not think about the requirements of the question because, "in maths you have to remember," but how was she supposed to remember when the question did not contain any clues? Other students also described the difficulties they experienced when the clues or cues they were used to were absent:

G: You can get a trigger, when she says like simultaneous equations and graphs, graphically, when they say like . . . and you know, it pushes that trigger, tells you what to do.

JB: What happens in the exam when you haven't got that?

G: You panic. (Gary, AH, Year 10, Set 3)

In the mock examination, some of the teachers even gave students the cues they needed to answer the questions:

L: My mind just went totally blank and I was really scared, a total blank, and I just couldn't focus, my concentration went completely and I just sat there like this . . . and I asked a question and said can you read it to me and explain a bit more and, without breaking the regulations she told me what it was about and I went, *oh, yeah* I remember now . . . and afterwards Miss Neville said to me you *know* that and—well sometimes you just need something to give you that little push, something to make you twig *(understand)* what it's about. (Liam, AH, Year 10, Set 3)

Liam's teacher told him after the examination that he knew that—because Liam knew how to operate the procedure, but he did not know which procedure to use or why. All of the students interviewed in Year 10 were convinced that this was an important problem—they could not interpret the demands of the examination questions. They knew mathematical rules and procedures, but they could not make use of them. Some of the students described this as being unable to apply their mathematics, some talked about the absence of cues, and others talked about not knowing the procedure to use. Yet they were all describing different aspects of the same problem; they could not use the methods they had learned unless the requirements of questions were explicit:

> 'Cause you haven't got a book (. . .) and so you've got to think of it and you think of it, but you think—but it could be, and then you think of about 20 different things it could be and you've got to decide which one. (Sara, AH, Year 10, Set 3)

The students' responses to the examination seemed to be consistent with the mathematical behavior they demonstrated in the assessments reported in chapter 6.

Using Mathematics in the Real World

When the students were in Years 9 and 10, I asked all of the students I interviewed ($n = 76$) to think of situations when they used mathematics outside school and to tell me whether they made use of school learned methods in these situations. The Amber Hill students, like the adults observed in other research settings (Lave, Murtaugh, & de la Rocha, 1984; Masingila, 1993; Nunes, Schliemann, & Carraher, 1993), all said that they abandoned school mathematics and used their own methods:

JB: And when you use maths in situations outside of school do you use the methods you have learned in school or do you tend to use your own?
D: You use your own.
S: Yeah you use your own. (Scott and Dean, Amber Hill, Year 9, Set 4)

S: I use my own methods
JB: Why?
S: It's easier, 'cause I know how to do it myself then don't I? it makes more sense. (Sacha, Amber Hill, Year 10, Set 4)

P: No, you use your own methods.
D: Yeah, your own methods. (Danielle and Paula, Amber Hill, Year 9, Set 2)

Previous research on the way in which adults have used mathematics in different settings has demonstrated that adults were unable to use much of the mathematics they learned in school in real-world situations (Lave, 1988). These students suggest that they could not use the methods they had learned in school in real-world situations even when they were still at school. This is probably not surprising given that students said they could not remember the mathematics they had learned a few weeks after learning it, when they needed to use it in another chapter of their books, in the same social situation with similar mathematical demands. Yet the students did not only choose their own methods over their school-learned methods because they could not remember or use school-learned mathematics. They chose not to use school-learned methods because of the way they interpreted the demands of the real world. The Amber Hill students believed the mathematics they encountered in school and the mathematics they met in the real world to be completely and inherently different. When I asked the students whether they believed the demands of the classroom and the real world presented any similarities, they all reported that they did not:

JB: When you use maths outside of school, does it feel like when you do maths in school or does it feel. . . .

K: No, it's different.

S: No way, it's *totally* different. (Keith and Simon, Amber Hill, Year 10, Set 7)

The students analyzed these differences in interesting ways:

J: They seem more important, worth doing, the things you do outside of school.

JB: Why is that?

J: Because you are doing it for yourself. (John and Paul, Amber Hill, Year 9, Set 1)

G: I use my own methods.

S: Yeah.

JB: Why is that do you think?

G: 'Cause when we're out of school yeah, we think, when we're out of school it's social, you're not like in school, it tends to be social, so it would be like too much change to refer back to here. (George, Amber Hill, Year 9, Set 3)

R: It's different 'cause you're like, you're doing it your own way and you're relying on yourself to get it right.

D: Yes I think it's different 'cause, like he says, you do it in a different way. (Richard & David, Amber Hill, Year 10, Set 2)

S: It's different 'cause you have to work it out for yourself, like, you haven't got a book to show you what you've got to do. (Shaun, Amber Hill, Year 10, Set 1)

The clarity of the students' perceptions on this issue is quite striking. Although there was no clear consensus about the reasons for the differences between mathematics in and out of school, all the students interviewed believed that using mathematics within school was a different experience from using mathematics outside school. Furthermore, the students gave reasons for their ideas of difference, which were close to the ideas proposed by various researchers in the field. George, in Set 3, was particularly interesting because he cited the influence of the social situation as the reason for his use of his own methods in preference to school methods. Lave (1988) has noted the influence of the social situation over adults' choice of methods, but George was not only influenced by the social situation—he was also aware of this influence. This suggests that his ideas of meaning and understanding in mathematical situations were strongly influenced by the social nature of the settings. His statement that "you're not like in school, it tends to be social, so it would be like *too much change* to refer back to here" gives a clear indication that his perceptions of the environments created by the real world and the mathematics classroom were inherently different.

John and Paul in Set 1 also concur with researchers such as Cobb (1986) when they say: "They seem more important, worth doing, the things you do outside of school." These students were able to cite the influence of their motivational goals on their choice of method, which again suggests that these goals had a strong influence upon them. It was clear from these students' descriptions that their use of mathematics in situations within and outside school was goal-driven and the goals formed were not inherently mathematical. Students described the importance of situations outside school, the lack of complication, the social nature of the real world and being alone, without books or teachers to help them. These differences caused the students to abandon their school-learned methods.

However, although these students showed that they did not make use of school methods in out-of-school situations, they were at least able to think for themselves and invent their own methods. Other students at Amber Hill painted a bleaker picture of their use of mathematics, indicating that their mathematical learning had disempowered them in more insidious ways, even stopping them from inventing their own methods:

JB: When you use maths outside of school, do you feel the same way as when you are doing maths in school or do they feel different?

J: They feel a lot different, like, um, you sort of have a little bit of understanding when you're in your lessons but your mind goes totally blank when you're outside.

JB: Why is that do you think?

J: You're not around people that understand it, like that can explain it to you and you're just like on your own . . . and you haven't got your little book with your notes. (Jackie, Amber Hill, Year 10, Set 1)

Schoenfeld (1992) lists seven typical student beliefs, one of which is that "the mathematics learned in school has little or nothing to do with the real world" (Schoenfeld, 1992, p. 359). The Amber Hill students' views seemed to concur with this assertion. These views clearly limited the usefulness of their school-learned mathematics, and I continue my analysis of the different reasons for this in the next chapter. Before doing so, I consider the responses of the Phoenix Park students to the different assessments reported in the last chapter and to their GCSE examination.

PHOENIX PARK

Performance Patterns

The last chapter's results provide some indication that the Phoenix Park students were at least as capable in test situations as the Amber Hill students. In long-term assessments, many more of the Phoenix Park students who had learned mathematics via projects were able to answer questions correctly 6 months after their lessons. The difference in performance between the students at the two schools and the difference between Phoenix Park students in Years 9 and 10 on the long-term assessments indicates that this was due to the way in which students had learned their mathematics. When the students were introduced to standard methods and procedures that they practiced, rather than used, they did not remember many of the procedures 6 months later. The students who had forgotten the largest proportion of their work (the Amber Hill Year 8, Set 1 students) were introduced to their methods at a fast pace, which was probably a contributory factor. The only learning that seemed to have been moderately successful in the long term was that of the Phoenix Park Year 10 students, who learned about estimation and statistics when they used these ideas within an applied activity.

In the two applied assessments, the Phoenix Park students did not demonstrate the particular problems that the Amber Hill students demonstrated, and the difference in performance of the students at the two schools became more marked as they experienced more of their different school approaches. In Year 8, many of the students at the two schools demonstrated a similar ability to solve problems related to angle and volume, apart from a significant proportion of the high set Amber Hill students who did not appear to interpret the demands of the situation well. In Year 9, the differences were more striking, and the Phoenix Park students were significantly more able to produce careful and accurate flat designs that incorporated their knowledge of measurement and scale and then successfully solve problems related to angle and area. They also demonstrated a freedom in approach that the Amber Hill students did not seem to possess. The Phoenix Park students' enhanced success derived from a capability and willingness to think mathematically in different situations and interpret the demands of varied settings, as I explore now.

Interpreting the Demands of Situations

At Phoenix Park, the students were interviewed in Year 10 a few weeks after completing their mock GCSE examinations. At this time, the students had experienced a few weeks of their examination preparation approach. This meant that projects had been abandoned and students had moved to a more formal and procedural system of learning. In interviews, the students reported that they found the GCSE mock examination difficult, but the students' concerns, which were reported in the last chapter, were completely different from those expressed by the Amber Hill students. The students were concerned that the examination included mathematical notation and content areas they had not met before, their projects were difficult to review, they did not receive any pressure to review, and, for some of them, they did not have calculators. Despite the differences between the nature of the students' project work and the GCSE examination, the Phoenix Park students did not report that they could not apply the methods they had learned, nor that they could not interpret the questions when they did not contain clues. Rather, the students reported that when they had learned the mathematics assessed they were able to make use of it:

JB: How did you get on in your mocks?
H: OK, it wasn't really hard.
JB: Did you find that the questions were different to what you were used to?
H: Well a lot of the stuff we hadn't done, until now, that's what we're doing now.
JB: And when you came across a question where it was something you had done, did you feel you were able to do the question?
H: Yes, I found it easy. (Hannah, PP, Year 10, JC)

The Amber Hill students who were given similar interview questions responded differently:

JB: And what about the questions that you *could* remember doing, when you recognized what to do, did you feel able to do those questions?
G: I still couldn't do them, because they were different, I couldn't apply the methods properly. (Carly, Amber Hill, Year 10, Set 1)

The Phoenix Park students' reports given in chapter 6 show that they faced a number of disadvantages that may have diminished their examination performance, but they still attained higher grades than the Amber

Hill students. The reason for this appeared to be that students could make use of the mathematics they had learned when it was assessed. Although they had not covered everything they needed for the examination, they could make effective use of the mathematics they had encountered before. The Phoenix Park students' superior performance on conceptual questions also provides an important clue as to the reason for their general success. The students were able to use mathematics in different situations because of their attitudes toward and beliefs about mathematics. When the students approached questions, they believed that they should consider the situations presented and interpret what they needed to do:

JB: Can you tell me about anything you like about maths?

T: I think it allows . . . when you first come to the school and you do your projects and it allows you to think more for yourself then when you were in middle school and you worked from the board or from books.

JB: And is that good for you do you think?

T: Yes.

JB: In what way?

T: It helped with the exams where we had to . . . had to think for ourselves there and work things out. (Tina, PP, Year 10, RT)

The students were not inhibited in the way the Amber Hill students were. They did not struggle to remember specific procedures, nor search for cues that might indicate the procedures to use. They were free to consider the different questions and make sense of them:

JB: Did you feel in your exam that there were things you hadn't done before?

A: Well, sometimes I suppose they put it in a way which throws you, but if there's stuff I actually haven't done before I'll try and make as much sense of it as I can, try and understand it and answer it as best as I can, and if it's wrong, it's wrong. (Angus, PP, Year 10, RT)

The Phoenix Park students were willing to try and think mathematically about questions and work out what was needed. This willingness appeared to derive from their belief in the value of thought in mathematics. Unlike the Amber Hill students, they did not believe that mathematical success depended on learning different procedures:

JB: Is there a lot to remember in maths?

S: There's a lot to learn, but then you need to know how to understand it and once you can do that, you can learn a lot.
P: It's not sort of learning is it? It's learning how to do things.
S: Yes, you don't need to learn facts, in the beginning of the maths paper they give you all the equations and facts you need to know. (Philip & Simon, PP, Year 10, JC)

This extract is particularly important. Lave (1996a) has claimed that notions of knowing should be replaced with notions of doing, arguing that the only indication that someone has knowledge is that they can use it. These students seemed to support this relational view of knowledge, as illustrated by the distinction drawn out by Paul: "It's not sort of learning is it? It's learning how to do things." This comment also highlights the difference between the Amber Hill and Phoenix Park approaches. At Amber Hill, teachers tried to transmit knowledge to students; at Phoenix Park, the students "learned how to do things" – they were required to use mathematics to solve problems. There was a marked contrast between the beliefs of these students and the Amber Hill students, who thought they needed to remember a vast number of rules and procedures. This difference in *belief* had an important impact on the students' use of mathematics in the GCSE examination and in the applied assessments. The Phoenix Park students did not feel the need to remember all the methods and procedures they had met:

JB: How long do you think you can remember work after you've done it?
G: Well I have an idea a long time after and I could probably go on from that, I wouldn't remember exactly how I done it, but I'd have an idea what to do. (Gary, PP, Year 10, MC)

Here Gary also supported a relational view of knowing; he dismissed the view that certain pieces of knowledge existed in his head that he could transfer from one situation to another ("I wouldn't remember exactly how I done it") and stated that his thoughts would only be *informed* by previously held ideas. He would "go on from that" and form new ideas of what he had to do in different situations. The Phoenix Park students only needed to remember an idea and move on from that, which may not have been as difficult as trying to remember a complex set of algorithms and procedures. This would also fit with the superior performance of the Year 9 students using statistics over the Year 8 students trying to recall a long division algorithm in the long-term learning tests. At Phoenix Park, the students seemed to have developed the disposition to think holistically about the requirements of situations probably because they needed to do this in their projects. They were prepared to think about questions even if they did not know or remember any set procedures to use. This approach will probably have contributed toward their superior performance on the conceptual questions in the examination and on applied and long-term assessments. The equivalent performance of both sets of students on procedural GCSE questions—despite the Amber Hill students' motivation, examination preparation, and commitment to learning procedures—was also due to the willingness of the Phoenix Park students to think for themselves and work out what they needed to do in procedural questions.

Using Mathematics in the Real World

At Amber Hill, the students reported that they did not make use of their school-learned mathematical methods because they could not see any connections between the mathematics of the classroom and the mathematics they met in their everyday lives. At Phoenix Park, the students did not regard the mathematics they learned in school as inherently different from the mathematics of the real world:

> JB: Can you think of a time outside school when you've had to do something mathematical ever?
> T: I do sometimes when I'm at home and I have to work out like prices and stuff, that's when I use it.
> JB: And is it similar or different to the way you do maths at school?
> T: Similar.
> L: Yes and sometimes you use it in other lessons in school, like in IT you use it sometimes. (Tanya and Laura, Phoenix Park, Year 9, MC)

> JB: When you do something with maths in it outside of school does it feel like when you are doing maths in school or does it feel different?
> G: No, I think I can connect back to what I done in class so I know what I'm doing.
> JB: What do you think?
> J: It just comes naturally, once you've learned it you don't forget. (Gavin and John, Phoenix Park, Year 9, MC)

When I asked the Phoenix Park students the same questions as the Amber Hill students about their use of school-learned methods or their own methods, three quarters of the 36 students chose their school-learned methods; this compared with none of the 40 Amber Hill students:

JB: And when you use maths in situations outside of school do you use the methods you have learned in school or do you tend to use your own?

T: Use those maths what I've learned here. (Tina, Phoenix Park, Year 10, RT)

A: What we've learned here probably has been helpful and I would probably look back and use that. (Angus, Phoenix Park, Year 10, RT)

G: I'd probably try and use what I've learned in school.

I: So would I. (Ian & Gary, Phoenix Park, Year 10, JC)

D: Probably try and think back to here and maybe try and think of my own methods sometimes, depending what sort of situation.

JB: So you would think back here for some things?

A: Yes it would be really easy to think back here.

JB: Why do you think that?

A: I dunno, I just remember a lot of stuff from here, it's not because it wasn't long ago, it's just because—it's just in my mind. (Danny & Alex, Phoenix Park, Year 10, JC)

The students also reported that they made use of their school-learned mathematics in a variety of different situations:

JB: Can you think of a time when you've used maths when you've been out of school?

G: Yes.

JB: What sort of situation?

G: My job at the Co-op.

JB: And you use maths there?

G: Yes.

JB: Do you find that you can?

G: Yes, it's easy. (Gary, Phoenix Park, Year 10, MC)

JB: Can you think of a time in your everyday lives when you've had to use something mathematical, any sort of maths?

I: I think a lot of the time you use it without noticing. (Ian, Phoenix Park, Year 9, RT)

N: Maths is a bit like integrated humanities.

JB: Why?

N: Because we use maths things there and humanities things here. (Nicola, Phoenix Park, Year 9, RT)

Although the students at the two schools were only giving their reports of their use of mathematics, these reports were consistent with the mathematical behavior they demonstrated in other situations. The Amber Hill students' descriptions indicated that they saw little use for the mathematics they learned in school in out-of-school situations. Hence, in real-world mathematical situations, they abandoned their school-learned mathematics and invented their own methods. The Amber Hill students appeared to regard the *worlds* of the school mathematics classroom and the rest of their lives as inherently different. This was not true for the Phoenix Park students, who had not constructed boundaries (Siskin, 1994) around their school mathematics knowledge in quite the same way. This idea is developed further in chapter 8, where I propose that the differences between the ideas and understandings of the Amber Hill and Phoenix Park students were indicative of two different *forms* of knowledge and that these differences derived directly from the teaching approaches they experienced.

8

Knowledge, Beliefs, and Mathematical Identities

In this chapter, I propose that the contrasting teaching and learning experiences of the students at Amber Hill and Phoenix Park schools produced some interesting and important differences in the students' mathematical knowledge and in the beliefs they held about mathematics. Further, I propose that these differences affected the students' relationship with the discipline of mathematics in important ways, giving the students different identities as learners and users of mathematics (Wenger, 1998).

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Whitehead (1962) proposes that it is possible to develop knowledge that is inert—that can only be recalled when it is specifically asked for. Schoenfeld (1985, 1988) asserts that students develop this type of knowledge in response to conventional pedagogic practices in mathematics that demonstrate specified routines that should be learned. These practices, he suggests, cause students to develop a procedural knowledge they can use only in standard textbook situations. In less procedural situations, students are then forced to base their mathematical decision making on irrelevant features of questions, such as the format in which they are presented or the key words used (Schoenfeld, 1988). The behavior described by Schoenfeld and the knowledge described by Whitehead (1962) seem to characterize the Amber Hill students' responses to different mathematical demands. It seemed that many of the students had developed an inert, procedural knowledge, and the reason for this was that the students had learned the teachers' methods and rules without really understanding them. This meant that in real or applied situations, the students were forced to look for cues that might indicate what they had to do.

The Amber Hill teachers encouraged students to learn the set methods they gave them because they thought this would make the subject clearer and easier for students. The students would not need to interpret the situations and understand what was going on as long as they could remember a procedure they had learned. When the teachers prepared the students for the examinations, they encouraged them to rehearse the rules they had taught them, rather than think mathematically about the situations presented:

> M: It's different to when you read them in the book, like he told us, sir told us that in our exam we don't look at the story, we just look at the numbers. (Marco, AH, Year 10, Set 4)

In the examination and applied assessments, students were forced to look for cues because they had no other way of knowing what to do. They were not prepared to interpret the mathematical demands of the situations, and they had not learned what different procedures meant or how they might adapt or change them if necessary. They did not know which procedures to choose, nor whether they were effective or correct having chosen them:

> S: You've got to—just like a computer, you'll do it, but when you get the answer you won't be sure that it's right, if it's like, you'll be like—this is how we learnt it, but is this the answer? you're never certain. (Simon, AH, Year 10, Set 7)

> JB: Could you do the questions?
>
> S: No, I couldn't, sometimes you can, but when it comes to really complicated ones you forget it and then you have to ask the teacher to go over it again and you think—I remember all this but you don't really remember what the point was. (Suzy, AH, Year 10, Set 2)

Both of these comments seem important. Belencky, Clinchy, Goldberger, and Tarule (1986) propose that some students develop relationships as received knowers—expecting to accept knowledge from authoritative sources without contributing to knowledge or making connections across areas. Simon seems to have developed such a relationship, saying that he reproduced the work as he had learned it, but "is this the answer? you're never certain." Suzy adds an important insight into the problem: The students remembered what to do, but they did not really remember what the point was. The Cognition and Technology Group at Vanderbilt ([CTGV] 1990) describe the ways in which different teaching approaches affect the perceptions students develop of mathematical concepts and procedures. They report that problem-oriented approaches to learning help students view mathematical concepts as useful tools that they can use in different situations. More traditional approaches to learning cause students to view concepts "as difficult ends to be tolerated rather than as exciting inventions (tools) that allow a variety of problems to be solved" (CTGV, 1990, p. 3). Brown, Collins, and Duguid (1989) draw similar distinctions between authentic and algorithmic approaches to teaching and the effect these have on the way students view mathematical concepts and procedures. The algorithmic approach experienced by the Amber Hill students caused them to view the procedures they had learned as abstract entities, useful only for solving school textbook questions. They did not hold the view that the algorithms they had learned were exciting and useful "inventions" that would give them the opportunity to solve different mathematical problems. The students' mathematics learning seemed to have created an important distinction in their minds between what they perceived as the algorithmic demands of school mathematics and the completely separate demands of the real world:

JB: When you use maths out of school, does it feel different to using it in school or does it feel the same?

R: Well, when I'm out of school, the maths from here is nothing to do with it to tell you the truth.

JB: What do you mean?

R: Well, it's nothing to do with this place, most of the things we've learned in school we would never use anywhere. (Richard, AH, Year 10, Set 2)

The Amber Hill students experienced difficulties using mathematics in the examination and applied assessments; they reported that school mathematics was unrelated to the mathematics problem they encountered in their jobs and lives, and when they did want to use methods many of them reported they were unable to apply them.

Resnick (1993) has suggested that many sociological theories lead to the belief that the main thing people learn in school is how to behave in school. This seemed to be true for the Amber Hill students: In lessons, the students tried to interpret what to do from the cues presented in questions and were often successful in doing so. In applied assessments, the students tried to do what was right, for example, demonstrating their knowledge of trigonometry in a question on angles, performing exact area calculations in a question on floor space. The students used the words *angle* and *area* as cues, rather than thinking holistically about the requirements of the questions. In the examinations, the students tried to interpret cues in a similar way, but found this to be difficult. In none of these situations did the students think mathematically; they did not think about the situations holistically or think about the mathematics to use. This was partly because of their perceptions about mathematics and partly because the students' learning was fixed, inflexible, and tied to the textbooks from which they had learned. It is because of this that I believe they could not use it in the real world. This is not to say that the students could not use mathematics outside school. As they reported, they invented their own methods in real situations and tried to work things out. But it does show that the mathematics they learned at school was of limited use in new and different situations. In the real world and in employment situations, the students would be left to learn on the job:

JB: So you've been doing roofing for about a year? There's quite a lot of maths involved in that isn't there?

R: Well, when I started that I was . . . when I got there, to be honest with you I was—what?? You know? It was like centimeters and inches and feet and angles and . . . like that, you know? And I was just—what?? But now I pick things up as I go along. (Richard, AH, Year 10, Set 2)

Given the motivation that the students demonstrated in their mathematics lessons and their beliefs about the importance of mathematics, this total lack of preparation for the mathematical demand of the real world must be considered unfair.

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The Phoenix Park students were considerably more confident in their use of mathematics in new and real situations than the Amber Hill students; they related this confidence to the approach of the school:

L: Yeah when we did percentages and that, we sort of worked them out as though we were out of school using them.

V: And most of the activities we did you could use.

L: Yeah most of the activities you'd use—not the actual same things as the activities, but things you could use them in. (Vicky & Lindsey, PP, Year 10, JC)

The students gave indications that the mathematics they learned through their project-based work was useful in new and different situations. This seemed to derive from a way of thinking and working in which the students learned to adapt and change methods to fit the demands of different situations. The students described the ways in which they developed meaning in interaction with different settings. Lindsey said she would use mathematics "not the actual same things as the activities, but things you could use them in"; she would adapt and transform what she had learned to fit new situations. Later in the interview she said:

> L: Well if you find a rule or a method, you try and adapt it to other things, when we found this rule that worked with the circles we started to work out the percentages and then adapted it, so we just took it further and took different steps and tried to adapt it to new situations. (Lindsey, PP, Year 10, JC)

The analysis offered by Lindsey in this extract is important because it was this willingness to adapt and change methods to fit new situations that seemed to underlie the students' confidence in their use of mathematics in real-world situations. Indeed many of the students' descriptions suggest that they had learned mathematics in a way that transcended the boundaries (Lave, 1996) that generally exist between the classroom and real situations.

> J: Solve the problems and think about other problems and solve them, problems that aren't connected with maths, think about them.
>
> JB: You think the way you do maths helps you to do that?
>
> J: Yes.
>
> JB: Things that aren't to do with maths?
>
> J: It's more the thinking side to sort of look at everything you've got and think about how to solve it. (Jackie, PP, Year 9, JC)

The idea that students may have developed a usable form of mathematics in response to their project work was partly supported by the students' views about the nature of their bookwork. When they described the mathematics they learned in SMP books at middle school, the mathematics they learned through their projects at Phoenix Park, and the examination preparation in year 10, the contrast they offered among the three approaches centered around the adaptability of their learning:

> JB: Do you think you learn different things—doing activities and working from a book?

L: I think you tend to understand it more when you do it with the activities.
V: 'Cause you're trying to work it out.
L: Yeah and you understand how they got it, when you're working from a book, you just know that's the thing and that you just stick to it, you tend to understand it more from the activities. (Vicky & Lindsey, PP, Year 10, JC)

JB: If you were in a job situation or something outside of school and there was something mathematical you had to do, do you think you would think back to things you'd learned here and use that?
L: I wouldn't be able to use the stuff now because I don't understand it [examination preparation].
H: No, we only understand it as in the way how, what it's been set, like this is a fraction, so all right then.
L: But, like Pope's theory I'll always remember—when you had to draw something I'll always remember the, like the projects we used to do.
H: Yeah, they were helpful for things you would use later, the projects. (Helen & Linda, PP, Year 10, MC)

When Helen says that "we only understand it as in the way what it's been set," she seems to be describing the inflexible nature of her learning, but she contrasts this with her project work, which she regarded "as helpful for things you would use later." Lindsey also seems to be describing the implicit boundaries that surround bookwork when she says, "you just know that's the thing and you just stick to it." Helen talks in similar terms: "Like this is a fraction, so all right then." As part of the Phoenix Park examination preparation, the students were taught to answer closed, procedural questions. They, like the Amber Hill students, regarded these rules as "set" and unchangeable. These descriptions contrast with Lindsey's earlier statement about project work: "Well if you find a rule or a method, you try and adapt it to other things."

There were a number of indications that the Phoenix Park students had developed a predisposition to think about and use mathematics in new and different situations, and this seemed to relate to a general mathematical empowerment. This empowerment meant that they were flexible in their approach and prepared to take what they had learned and adapt it to fit new situations. This flexibility seemed to rest on two important principles. First, the students believed that the mathematics they learned was adaptable. Many researchers have shown the rigid and inflexible models of mathematics that students develop that stop them from using mathematics in new situations (Brown, Collins, & Duguid, 1989; Schoenfeld, 1985; Young, 1993). The comments given by Phoenix Park students in chapter 5 demonstrate that they viewed mathematics as an active, exploratory, and adaptable subject. The second important feature of their learning was their ability to adapt and change methods and to think mathematically.

T: Yes, when I go shopping I just . . . get all the things in the basket, number them all at a pound, for example if some of them are 50p and some of them are £2, I just call them all a pound and see how much I've got in my pocket, then hope for the best. It usually gets me . . . it's worked every time actually.

JB: That's not a bad strategy if some of the prices are more and some less.

T: Yes, you've just got to make sure that there's more that are less than a pound than more than a pound or else you haven't got enough money. (Trevor, PP, Year 10, RT)

Trevor was not describing any complex mathematical thinking in this extract, but his description was interesting for two reasons. First, Trevor chose this situation as an example of the way he used his school-learned mathematics in the real world. Second, his statement demonstrates the confidence he had to think mathematically in a real situation. The students were clear in interviews about the source of their mathematical confidence. They related this to two features of their approach: The fact that they had been forced to become autonomous learners, and that they had always been encouraged to think for themselves.

N: You had to be self-motivated.

JB: Is that fair do you think?

N: Well, it was good for us because it taught us to do things by ourselves so it made you confident to do things for yourself. (Nicola, PP, Year 10, RT)

JB: Did doing the project work help you in any way do you think?

T: Yes, thinking for yourself and motivating yourself I think. (Tina, PP, Year 10, RT)

S: At the start of year 9 (8), the teacher told you what to do and explained all the skills and you just did it and then gradually you begin to think more for yourself—you know—what shall I do next?, what shall I do about this? (Simon, PP, Year 10, JC)

The students contrasted their experiences of project work and bookwork by saying that the former required them to work things out and think, whereas books that comprised only short procedural questions did not:

T: I think it allows, when you first come to the school you do your projects and it allows you to think more for yourself than when you were in middle school and you worked from the board or from books, things like that. (Tina, PP, Year 10, RT)

A: With the SMP books it just sort of . . . say you were doing the SMP books on percentages or something, it would just ask you a series of questions on it, like find the percentages of this and that, but if you did an investigation on it, you would have to like think a lot more about it for yourself and how to like solve the problem. I would say it's a lot more interesting than doing SMP books. (Angus, PP, Year 10, RT)

This requirement to think in mathematics lessons was central to Phoenix Park's approach. The students were given little structure and guidance and although many spent long periods of time off task, when they were working they needed to be thinking. It was almost impossible for the students to switch off and work in a procedural way when they were planning and developing their projects. For some students, this was the most important difference between their bookwork and project work:

G: In books it more or less explains everything to it, but I'd rather work it out by myself by looking at it and working it out or getting the teacher to talk to you about it, instead of telling you exactly what to do.
I: And in the books you don't understand it.
G: And you take it in if you've done it but if you read it, you just read it and you don't take any notice. (Ian & Gary, PP, Year 10, JC)

H: The stuff we're doing now, it's more fractions and figures [examination preparation].
L: Like we'll do a lesson or something and some of us don't understand it and then next lesson we'll do something completely different, that's harder and you can't remember anything.
JB: So what's different?
L: We were using them before, but now we're just writing them.

H: And vaguely understanding them and having a little bit of discussion and thinking oh I don't understand that or I understand this and then you just leave it, but I'd say some of the work we did before we do use now or out of school or whatever. (Linda & Helen, PP, Year 10, MC)

For Linda, the difference between bookwork and project work was, "we were using them before, but now we're just writing them." Gary drew out a similar distinction, saying, "you take it in if you've done it, but if you read it you just read it." Helen chose to say, without prompting or asking, that she used the mathematics she learned through the projects "now or out of school or whatever." Perhaps the most important distinction of all between project work and the closed books they worked on was provided by Sue. I asked her the following question at the time when students had finished their project work and were preparing for the national examination by practicing procedures in closed questions:

JB: Do you think when you use maths outside of school, it feels very different to using maths in school, or does it feel similar?

S: Very different from what we do now, if we do use maths outside of school it's got the same atmosphere as how it used to be, but not now.

JB: What do you mean by—it's got the same atmosphere?

S: Well, when we used to do projects, it was like that, looking at things and working them out, solving them—so it was similar to that, but it's not similar to this stuff now, it's, you don't know what this stuff is for really, except the exam. (Sue, PP, Year 10, MC)

Sue was particularly lucid in her comparison of the two approaches: One was about solving problems, "looking at things and working them out, solving them"; the other did not hold any meaning for her—"you don't know what this stuff is for really, except the exam." Sue, like other students, distinguished between the two approaches in terms of the usefulness of the mathematics she had learned. One version was similar to the mathematics of the real world, whereas the other was not.

MATHEMATICAL IDENTITIES

The relative underachievement of the Amber Hill students in formal test situations may be considered surprising partly because the students worked hard in mathematics lessons and partly because the school's mathematical approach was meant to be examination-oriented. However, after many hours of watching the Amber Hill students work and talking to the students in interviews, I was not surprised by the relative performance of the two sets of students. This was because the learning of the Amber Hill students was extremely inflexible and inert; although many were able to use their mathematical methods within textbook questions, when the demands of situations were only slightly different than the ones they were used to, they failed. The students had developed a knowledge that appeared to be effective only in other textbook or similar situations.

One way to interpret these results would be to say that the Phoenix Park students learned more than the Amber Hill students, but such an analysis would be misleading and incomplete. That is because the Amber Hill students learned a lot during their time in their mathematics classrooms—they learned to become extremely successful participants of their school mathematics classrooms, interpreting subtle classroom cues and gaining success as they worked through their exercises. The students only experienced problems because this effectiveness did not transfer elsewhere, as one of the students described:

> A: It's stupid really 'cause when you're in the lesson, when you're doing work—even when it's hard—you get the odd one or two wrong, but most of them you get right, and you think well, when I go into the exam I'm gonna get most of them right, 'cause you get all your chapters right. But you don't. (Alan, AH, Year 10)

Being effective in the classroom involved strict adherence to school and mathematical rules, interpretation of nonmathematical cues, and suppression of thought. All of these practices became part of the students' mathematical learning identities. These practices were not deliberately encouraged by the Amber Hill mathematics teachers, but they gradually developed among the community of learners, and practices that were rare at the beginning of Year 8 were well established by the end of Year 10. The relationships that students developed with mathematics had enormous implications for the students' use of mathematics in the real world. The students reported that in their jobs and everyday lives, they faced completely different demands than those of the classroom, and they had not developed the idea that they could adapt what they had learned to new situations. The students did not regard themselves as mathematical problem solvers. This meant that when they met new situations, they abandoned the mathematics they had learned in school and were forced to rely on their own invented methods. The realization of this difference between the environments of school and the real world caused the students to believe school mathematics to be an irrelevance, and they developed the idea that their school mathematics knowledge had boundaries (Siskin, 1994; Wenger, 1998) or barriers surrounding it, which kept it firmly within the mathematics classroom. When I asked 40 of the students if they used mathematics outside school, they all said they did. When I asked the students if they made use of methods they learned in school, they all said they did not. The students talked about their perceptions of school mathematics as a strange and specialized type of code that they would only use in one place—the mathematics classroom. What was particularly interesting was the fact that students all offered different reasons why they would not make use of school mathematics, but these all related to the differences between the *environments* of school and the real world. The students did not describe mathematical differences or the problems of their mathematics teaching, which they were happy to talk about at other times. Rather, they described the importance of situations outside school, the lack of complication, the social nature of the real world, and being separated from the partners with whom they worked in lessons. For example:

G: I use my own methods.

JB: Why is that do you think?

G: 'Cause when we're out of school yeah, we think, when we're out of school it's *social*, you're not like in school, it tends to be *social*, so it would be like too much change to refer back to here.(George, Amber Hill, Year 10, Set 3)

George's idea that referring back to school would be "too much change" is important because it suggests that the lack of discussion and negotiation he experienced in Amber Hill classrooms not only denied him opportunities to develop understanding of mathematics, but set up key ideas about difference. George regarded the individualistic mathematics classroom as fundamentally different from the socially constituted real world. The differences the students described, which caused them to abandon their use of school-learned methods, related to the constraints and affordances of the real world and the presence of different people and systems in the environment. It was clear from the students' descriptions that their use of mathematics in situations within and outside school was driven by the different environments. Although their individual cognitive attributes were important, these were not the only factors influencing their use of mathematics.

Psychological theories of learning have been dominant within mathematics education since its inception as a research domain. Yet we are now, as Resnick (1993) has claimed, in the midst of attempts to merge the social and cognitive (Schoenfeld, 1999). Situated perspectives on learning have made a particularly significant contribution over recent years, presenting knowledge not as a stable, individual characteristic, but something distributed between people and the activities and systems of their environment (Brandsford, Brown, & Cocking, 1999; Lave, 1988, 1996). Learning in the situated perspective becomes a process of changing participation in changing communities of practice (Lave & Wenger, 1991), and a person's knowledge ability is regarded to be a function of the environments in which he or she operates. Such perspectives allow for knowledge variance between contexts and situations, which many researchers have observed to be a characteristic of human behavior (Lave, 1988; McDermott, 1993; Säljö & Wyndhamn, 1993). Within mathematics education, the classroom community, including the implicit and explicit norms and practices that prevail, becomes extremely important—not as a vehicle for learning, but an intrinsic part of the knowledge that is generated and used. Theories of situativity may be characterized by their "focus on interactive systems that are larger than the behavior and cognitive processes of an individual agent" (Greeno & MMAP, 1998, pp. 5–6). Students do not just learn methods and processes in mathematics classrooms; they learn to *be* mathematics learners, and their learning of content knowledge cannot be separated from their engagement in the classroom because the two mutually constitute one another at the time of learning. The importance of this interaction has not been fully recognized in mathematics education, and researchers are only now beginning to realize that different pedagogies are not just vehicles for more or less knowledge, they shape the nature of the knowledge produced and define the identities students develop as mathematics learners through the practices in which they engage.

Wenger (1998) proposes that learning is a process of identity formation, and that students locate themselves within particular communities of practice in a process of belonging and, ultimately, knowing:

> Because learning transforms who we are and what we can do, it is an experience of identity. It is not just an accumulation of skills and information, but a process of becoming—to become a certain person or, conversely, to avoid becoming a certain person. Even the learning that we do entirely by ourselves contributes to making us into a specific kind of person. We accumulate skills and information, not in the abstract as ends in themselves, but in the service of an identity. (Wenger, 1998, p. 215)

A situated perspective on learning does not imply that certain teaching practices are better than others, but it does suggest that the activities of different practices are central to what is learned. Thus, Greeno and MMAP (1998) assert that classroom discussions are important not only as a way of making content meaningful, but because students are learning to participate in discourse practices. Similarly, project work is important as students "develop abilities of collaborative inquiry and of using the concepts and methods of a discipline to solve problems" (p. 15), and representational systems are valuable because they enable students to learn to use and appreciate different systems of representation. The reflections of the Amber Hill students suggest that the *practices* in which they commonly engaged in their classrooms were not present elsewhere, and this difference caused them to abandon their school-learned methods as soon as they left the classroom.

Dreyfus (1984) describes the development of expert knowledge as moving through five stages—from novice to expert. The different stages relate to the degrees of affiliation or involvement that learners have. In the novice stage, knowledge is mediated by rules and maxims, but as students learn more, they develop more agency and responsibility and start to form their own questions and devise moves in response to them. All of the students interviewed appeared to regard themselves as rule followers rather than active agents, suggesting that they had not moved beyond the novice stage of learning. This is not surprising because traditional systems of school mathematics differ from both the practices of mathematicians and mathematics users in the world (Burton, 1999a, 1999b; Noss, 1994) precisely because of their emphasis on rule following and the limited opportunities for agency provided by such systems.

Traditional teaching methods have been challenged on many counts, with suggestions that they are not interesting and that they give students little opportunity to construct understanding. But situative perspectives add something new and different to the conversation because they focus on the classroom practices that define the knowledge produced, suggesting that practices of individualized, abstract procedure reproduction "deny students the chance to engage the relevant domain culture" (Brown, Collins, & Duguid, 1989, p. 34). Neither professional mathematicians (Burton, 1999a, 1999b) nor professional users of mathematics (Noss, 1994) spend their time reproducing standard procedures—that is a particular practice specific to the mathematics classroom. Yet the specificity of that practice may be the single most important factor reducing achievement and affiliation for students. The suggestion that mathematics teaching approaches should offer varied, realistic constraints and engage students in discussion and negotiation is far from new, but the situated perspective adds another dimension to such proposals. For if learning mathematics entails more than the construction of cognitive forms, but the development of practices through which identities with the discipline are formed, then repeated and limited practices of procedure repetition will limit the identities of all students who do not go beyond such practices. A classroom community that lacks the practices of mathematical problem solving may bound (Siskin,

1994) students' knowledge and give them passive identities as receivers, rather than users of mathematics knowledge. Thus, it is not the extent of knowledge that is in question, but its accessibility.

At Phoenix Park, the students were probably less motivated to do well in examinations. They spent less time on task in lessons, they had not been introduced to all of the content and procedures they needed in the examination, and they were not even given the necessary equipment for the examination. But the students were more successful in the examination, and in applied and long-term assessments, apparently because they had developed more effective forms of knowledge and more productive identities as mathematics learners. They were able to use mathematics in a range of different situations; and the reason for this was not because they had learned it in a clear and straightforward way, but because they had used mathematics in a similar way in the classroom. They had engaged as mathematical problem solvers, and they were willing to consider any mathematical situation and think about what was involved. As Sue said, when they used mathematics outside school, it felt the same—it "had the same atmosphere" as their project-based mathematics. Even the vast differences between the nature of Phoenix Park's open-ended projects and the GCSE examination did not faze the students. This was because they did not regard the two assessments as inherently different:

> JB: Did the questions in the exam seem similar to what you'd done in class or did they seem different?
>
> L: Most of them seemed similar didn't they?
>
> JB: The exam was similar to your project work?
>
> L: Most of it seemed the same really. (Louise, PP, Year 10, JC)

Within school, the Phoenix Park students did not view mathematics as a formalized and abstract entity that was useful only for school mathematics problems. They had not constructed "boundaries" (Siskin, 1994) around their school mathematical understandings in the way that the Amber Hill students had. At Amber Hill, the students developed a narrow view of mathematics that they regarded as useful only within classroom textbook situations. The students regarded the school mathematics classroom as one "community of practice" (Lave, 1993, 1996) and other places, even the school examination hall, as different communities of practice.

Lave (1996a) claims that learning would be enhanced if we were to consider and understand how barriers are generated that make individuals view the worlds of school and the rest of their lives as different communities of practice. At Amber Hill, there were strong institutional barriers that separated the students' experiences of school from their experiences of the rest of the world. Many of these barriers were constituents of Bernstein's visible pedagogy (Bernstein, 1975). General school rules and practices such as school uniform, timetables, discipline, and order contributed to these as well as the esoteric mathematical practices of formalization and rule following. At Phoenix Park, the barriers between school and the real world were less distinct: There were no bells at the school, students did not wear uniforms, teachers did not give students orders, students could make choices about the nature and organization of their work and whether they worked or not, mathematics was not presented as a formalized, algorithmic subject, and the mathematics classroom was a social arena. The communities of practice making up school and the real world were not inherently different. From this perspective, the Phoenix Park students were more able to make use of their school-learned mathematics because they had been enculturated into a practice of thinking, talking, representing, and interpreting in the classroom, and they had developed productive relationships with the discipline of mathematics. The students' knowledge of mathematical procedures at the two schools may have been similar, but the way they connected and interacted with mathematics and formed mathematical relations was different because of the practices in which they engaged in school and the effect of those practices on the mathematical identities students developed.

When the students were presented with the angle problem in the architectural task, many of the Amber Hill students were unsuccessful not because they were not capable of estimating an angle, but because they did not consider the situation or interpret it correctly. Similarly, in the flat design task, 25 Amber Hill students did not work out the area of their flat not because they were incapable of calculating areas, but because they did not interpret what was needed in the situation. The Phoenix Park students, in contrast, were not as well versed in mathematical procedures, but they were able to interpret and develop meaning in the situations they encountered. The fact that the Amber Hill students had learned more procedures than the Phoenix Park students demonstrates the inadequacy of transfer theories in explaining individuals' use or nonuse of subject matter. This is because the Amber Hill students' nonuse of mathematics had nothing to do with the knowledge of procedures they did or did not own—the students could reproduce relevant procedures in *certain* contexts. Similarly, the Phoenix Park students' effective use of mathematics must be taken as a support for a relational view of learning because it was the students' ability or predisposition to think and form meaning in different settings that differentiated them from the Amber Hill students. The similarity in some of the procedures the students knew, alongside the vast differences in the way they used the procedures, illustrate the need for theories of learning to go beyond cognitive representations of knowledge to consider the ways that knowledge plays out in the world and the relationships students develop with their knowledge. This suggests that effective teaching practices need to do more than provide students with memorized knowledge; they must enable students to develop productive dispositions and relationships with the discipline they are learning (Boaler, 2000a, 2002c).

It is not the case that *all* the Amber Hill students had developed a shallow, procedural knowledge and that *all* the Phoenix Park students were able to use mathematics effectively. At Phoenix Park, some of the students persisted in the belief that they needed to learn set rules and methods to be successful mathematicians. Others took to the open approach and flourished within it. I did not find out why different students responded to the Phoenix Park approach in different ways. Nevertheless, it is not surprising that some students resisted the open nature of the mathematics when they had learned mathematics in a different way for 8 years prior to attending Phoenix Park and they were only at Phoenix Park for 3 years. Similarly, at Amber Hill, some students developed an effective mathematical understanding because they were able to look beyond what they were given and make their own sense of the different methods they encountered. The two approaches are not at opposite ends of a spectrum of mathematical effectiveness, but the differences between the approaches do serve to illuminate the potential of the different methods of teaching for the development of different *forms* of knowledge and the cultivation of different identities as learners and users of mathematics.

9

Girls, Boys, and Learning Styles

> *Perhaps we don't take seriously enough the voices that say again and again, "but it doesn't make sense," and "what's the point of it?" Perhaps what they are saying simply is true. Perhaps mathematics, their mathematics, secondary-school mathematics, doesn't make sense. Perhaps the fault is in the mathematics, and not the teaching, not the learning, not the people. At the very least it is a question worth focusing on for a while.*
>
> (Johnston, 1995, p. 225)

Johnston presents an important idea in this extract that I intend to explore and develop in this chapter. Many of the Amber Hill students experienced a mathematics that made little sense to them and although both girls and boys were negatively affected by this, the greatest disadvantages were experienced by the girls mainly because of their preferred learning styles and ways of working. In this chapter, I examine the different responses of the girls and boys at the two schools. In doing so, I hope to extend theoretical positions about the learning styles of girls and boys and the potential of different approaches for equity (see also Boaler, 1997a, 1997b, 2002b). I also hope to further illuminate the different learning experiences of the Amber Hill and Phoenix Park students and the effect that these had on their understanding of mathematics.

The underachievement and nonparticipation of girls in mathematics has become an established focus of concern in recent years. As a result, many feminists and others with equity concerns have developed a range of initiatives that have been successful at raising girls' achievement, if not their continued participation. In England, girls now attain the same proportion of the top grades in the GCSE examination (i.e., A*–C) as boys, and stereotyped attitudes about the irrelevance of mathematics for girls are largely disappearing. However, important differences still occur among the top 5% of students in England, the United States, and many other countries (Leder, Brew, & Rowley, 1999). In 1993 in England, approximately 5 boys to every 4 girls attained a GCSE grade A, and girls made up only 35% of the 1995 A-level cohort. These differences that prevail among the highest attaining students are important.

One of the consistent themes that has emerged from the literature on gender differences has concerned open, problem-solving environments. These have been claimed to produce equity among students (Burton, 1995; Rogers & Kaiser, 1995), although there has been little research evidence available to support such claims. Official sources such as HMI (1985) and Cockcroft (1982) in England and the NCTM (1989, 2000) in the United States have also made proposals to further the use of open-ended work to improve the mathematical experiences of boys and girls. Where feminist researchers have diverged from the more general reformists is in their claim that school mathematics has traditionally disadvantaged girls because of the ways girls tend to think and work and the ways they come to *know*.

Becker (1995) and Belencky et al. (1986) both take Gilligan's (1982) notion of "separate and connected knowing" (p. 35) to suggest that women and men have differential preferences for ways of knowing and subsequent ways of working. Thus, women tend to value connected knowing, characterized by intuition, creativity, and experience, whereas men tend to value separate knowing, characterized by logic, rigor, and rationality. Becker (1995) claims that girls have traditionally been denied access to success in mathematics because they tend to be connected knowers, and traditional models of mathematics teaching have encouraged separate ways of working. Head (1995) has suggested that girls also prefer cooperative, supportive working environments, whereas boys work well in competitive, pressurized environments. These various claims about the gendered preferences of students seem important to consider in the light of the girls' and boys' responses to mathematics teaching at Amber Hill and Phoenix Park. This is because the Phoenix Park approach encouraged students to develop mathematics knowledge through open problems that seemed to enable both separate and connected thinking. The responses of girls to this, and the traditional approach of Amber Hill, inform the new theoretical positions proposed.

GENDER PATTERNS AT AMBER HILL

Throughout my research study, many of the girls *and* boys at Amber Hill expressed strong preferences for their coursework lessons and spoke vividly about their dislike of textbook lessons. However, the reasons the girls and boys gave for their preferences and, more important, the responses of the students to the textbook approach they disliked, were qualitatively different. This difference was intricate and complex, but for the girls it involved what I would call a *quest for understanding*; for the boys, it involved playing a kind of school mathematics *game*. I attempt to illustrate and illuminate these propositions now.

The Quest for Understanding

All of the Amber Hill girls interviewed in Years 9 and 10 expressed a strong preference for their coursework lessons and the individualized booklet approach, which they followed in Years 6 and 7, as against their textbook work. The girls gave clear reasons why these two approaches were more appropriate ways of learning mathematics for them; all of these reasons were linked to their desire to understand mathematics. In conversations and interviews, students expressed a concern for their lack of understanding of the mathematics they encountered in class. This was particularly acute for the girls not because they understood less than the boys, but because they appeared to be less willing to relinquish their desire for understanding:

J: He'll write it on the board and you end up thinking, well how comes this and this?, how did you get that answer? why did you do that?, but . . .

M: You don't really know because he's gone through it on the board so fast and . . .

J: Because he understands it he thinks we all do and we don't. (Jane and Mary, AH, Year 10, Set 1)

These students indicated that they were interested in meaning and understanding. They did not just want to know about different methods—they wanted to know "How comes this and this?" "How did you get that answer?" Many of the boys did not like their textbook lessons, and they did not understand more of the work than the girls, but they seemed to have formed different goals to the girls. These related to speed and the attainment of correct answers, rather than understanding. Thus, typically:

A: I don't mind working out of textbooks, because you can get ahead of everyone else. (Alan, AH, Year 10, Set 3)

J: I dunno, the only maths lessons you like are when you've really done a lot of work and you're proud of yourself because you've done so much work, you're so much ahead of everyone else. (James, AH, Year 9, Set 2)

Both of these boys emphasized the importance of *relative* performance (Head, 1995), rather than absolute learning. The goals and expectations of many of the boys related to working quickly and completing lots of questions. These were not particularly beneficial goals, in the long term, because the boys came to regard mathematics as a system of rule following and rote learning. However, as a coping strategy, the boys' response was more productive in accommodating to the demands of the school system. Many of the girls were concerned about understanding their mathematics; because they felt they were unable to do so, they would often become anxious and fall behind.

J: When I understand there's no stopping me, you saw me with that, when we had that equation sheet and the end of the lesson came and I was—do we have to go? I just want to finish this. Once I understand something I'm all right, but it kind of frustrates me if I'm sitting there for an hour and I don't know exactly what I'm doing. (Jane, AH, Year 10, Set 1)

M: The only work I like is when I understand what I'm doing, it's when I don't understand and I get confused, that's when I don't like it much. (Mary, AH, Year 10, Set 1)

As a result of a number of different data sources, I became convinced that it was this desire to understand, rather than any difference in understanding, that differentiated some of the girls from the boys. The girls knew they needed to understand mathematics, but they felt they had no access to understanding within their fast, pressured textbook system.

S: I just try and do it now, I don't know what it means, I just try and work fast. (Sara, AH, Year 9, Set 3)

The girls' preference for understanding caused them to become disaffected in relation to mathematics. For some girls, this disaffection was heightened by their awareness of the mismatch between their desire for understanding and their classroom experiences:

JB: Is maths more about understanding work or remembering it?

J: More understanding, if you understand it you're bound to remember it.

L: Yeah, but the way sir teaches, it's like he just wants us to remember it, when you don't really understand things. (Louise and Jackie, AH, Year 9, Set 1)

Further evidence of the different priorities held by the girls and boys at Amber Hill came from questionnaires. In their Year 9 questionnaire, I asked the students to rank five different areas of mathematics in terms of their importance. These were: (a) getting a lot of work done, (b) working at a fast pace, (c) understanding, (d) remembering rules and methods, and (e) knowing how to use a calculator. Three of these categories produced statistically significant differences between girls and boys at Amber Hill:

- 91% of girls regarded understanding as the most important aspect of learning mathematics, compared with 65% of boys.
- 4% of girls regarded remembering rules and methods as the most important, compared with 24% of boys.
- 5% of girls regarded getting a lot of work done as the most or second most important aspect of learning mathematics, compared with 19% of boys.

The differential responses of girls and boys were also evident in lessons. During my lesson observations, I frequently observed boys racing through their textbook questions trying to work as quickly as possible and complete as many questions as they could. Just as frequently, I observed girls looking lost and confused, struggling to understand their work or giving up all together. In lessons, I often asked students to explain what they were doing. The vast majority of the time, the students would tell me the chapter title and, if I asked them questions like "Yes, but what are you actually *doing*?", they would tell me the number in the exercise. Neither girls nor boys would describe the mathematics they were using, or tell me why they were using methods or what they meant. On the whole, the boys were content if they attained correct answers. The girls would also attain correct answers, but they wanted more:

> M: It's like, you have to work it out and you get the right answers but you don't know what you did, you don't know how you got them, you know? (Marsha, AH, Year 9, Set 4)

Marsha, like Jane and Mary earlier, demonstrates a desire for understanding and meaning that extends beyond the acquisition of right answers.

POSITIVE LEARNING EXPERIENCES

The girls at Amber Hill were not only critical of their school's mathematics methods; in interviews, they offered extremely clear descriptions of positive learning experiences. All of these experiences took place during coursework lessons or individualized booklet lessons. The reasons the girls liked these approaches were related to the freedom they experienced to either use their own ideas, work as a group, or work at their own pace. All these practices, the girls claimed, gave them access to a depth of understanding that textbook work denied them.

In chapter 4, I described the preferences students held for open-ended work. This was generally because the students did not believe that their textbook lessons allowed them to use their own ideas or think creatively. Preferences for these features of their learning—features that allowed a connected way of working (Becker, 1995; Gilligan, 1982)—were more prevalent among girls than boys. This was shown by some of the Year 10 questionnaire responses that prompted significant differences between girls and boys. Significantly more boys agreed with the following:

- "It is important in maths to answer questions the way the teacher wants you to" (girls = 49%, boys = 70%).

In contrast, significantly more girls agreed with the statements:

- "It is important in maths to find your own way of solving problems" (girls = 84%, boys = 66%).
- "It is important in maths to think about different types of maths" (girls = 87%, boys = 71%).

These responses seem important because larger proportions of girls were expressing preferences for a freedom of approach and a way of working that they rarely experienced in their mathematics classrooms.

Both boys and girls at Amber Hill reported enjoying their open-ended coursework, but the boys were less convinced of the value of having to think for themselves and the need to put effort into their work mainly because this conflicted with their desire for speed and correct answers:

G: I don't really like investigations.

JB: Why not?

G: It's hard.

JB: How are they different to what you do normally?

G: Because in chapters, the teacher explains how to do it, but with the investigations you have to do it by yourself.

JB: Is that more difficult?

G: Yeah, 'cause in the chapters, once you know how to do it, you're away. (Gary, AH, Year 10, Set 3)

Although many of the boys reported enjoying their coursework, this would generally be because it was a change. Few of the boys talked about the opportunity to think or to use their initiative, or the access it gave them to understanding, whereas this was central to the girls' reasoning.

The girls at Amber Hill also expressed preferences for working cooperatively in groups, but again the reason for this was the access that discussion and group work gave them to depth of understanding. The boys rarely mentioned their experiences of group work, and those that did differed in their responses to it. Some of the boys disliked working in groups because they felt that it slowed them down:

> L: Well, it could have been useful, but you could do it in half the time yourself, like you speed along, you understand it, next topic. But it slows you down, the rest of the class. (Leigh, AH, Year 10, Set 2)

The different responses of the girls and boys to group work related to the opportunity it gave them to think about topics in depth and increase their understanding through discussion. This was not perceived as a great advantage to the boys probably because their aim was not to understand, but to get through work quickly. These different responses were also evident in response to the students' preferences for working at their own pace. In chapter 6, I showed that an overwhelming desire for both girls and boys at Amber Hill was to work at their own pace. This desire united the sexes, but the reasons boys and girls gave for their preferences were generally different. The boys said they enjoyed individualized work that could be completed at their own pace because it allowed them to tear ahead and complete as many books as possible:

> C: It was better then weren't it?
>
> M: Yes.
>
> C: We used to compete.
>
> M: Yeah, we could do it at our own pace.
>
> C: Yeah, we could do it at our own pace and we used to be books ahead of the others. (Chris and Marco, AH, Year 10, Set 4)

> A: Before, when we had the little books, they were only short pages and we used to like compete with each other, see who'd done the most, who'd got the most percentage and that was like, most interesting. (Alan, AH, Year 10, Set 3)

The girls wanted to work at their own pace so they could understand what they were doing before they moved onto something else:

L: We had time to read it didn't we? We had time to read it through and if we didn't get it we had time to read it again, but like with this, we can only read it through once because she wants us to hurry up and get on and finish it. (Lindsey, AH, Year 10, Set 4)

The girls again explained their preference for working at their own pace in terms of an increased access to understanding. The girls at Amber Hill consistently demonstrated that they believed in the importance of an open, reflective style of learning, and that they did not value a competitive approach or one in which there was one teacher-determined answer. Unfortunately for them, the approach they thought would enhance their understanding was not attainable in their mathematics classrooms except for 3 weeks of each year.

Top Set Girls

In chapter 4, I described the speed and pressure that were an important part of the Set 1 experience at Amber Hill. Many of the students reported that these features of their Set 1 lessons had a negative effect on their learning, and this effect seemed to be particularly detrimental for the girls. In the top set group in my case study cohort (n = 33), I identified 15 students who were underachieving. This identification derived from a comparison of their NFER scores for mathematics on entry to the school and their success in Years 6 and 7 when they used SMP booklets with: their relative positions in the Set 1 group, my assessment exercises, their GCSE grades, and the opinion of their teacher. Eleven of the 15 students were girls, which represented over two thirds of the girls in the group. In the short contextualized questions given to students at the beginning of Year 8 and again at the end of Year 9, nine of these 15 students attained *lower* grades in Year 9 than in Year 8, whereas the rest of the top set improved their grades or stayed at the same level. Most of the 15 students were easy to locate in lessons. Six of the girls sat together and looked lost, confused, and unhappy in lessons and completed hardly any work. At one time, some of these girls were the highest mathematical attainers in the school. On entry to the school, Carly attained the highest NFER entry mark in the school, and Lorna attained the second highest mark; both of these girls attained the *lowest* GCSE grade in Set 1—grade E. In the Year 9 questionnaires, when students were asked to describe lessons, Carly and Lorna gave the following descriptions:

Carly: Not interesting. You go through the work too quickly and things don't get explained properly. (Carly, Amber Hill, Year 9, Set 1)

Lorna: The teacher stands by the blackboard for half the lesson explaining the work and everyone seems confused and not understanding the work. It goes too fast and it is very uninteresting. (Lorna, Amber Hill, Year 9, Set 1)

In the top set, there were 16 girls and 17 boys. In GCSE examinations, there were significant differences between the achievement of the girls and boys in Set 1 even among such a small number of students. In the GCSE examinations, boys attained 14 of the 19 A–C grades from Set 1; girls attained 11 of the 14 D and E grades. Gender differences in achievement were most marked among the highest attaining students in the school, which is consistent with national patterns of mathematics performance. Although such differences affect a small proportion of girls, they are extremely important because these high-attaining girls, who could and should be getting high grades, are the students who could be future role models, such as mathematicians, engineers, and teachers of mathematics. The girls are also being denied access to a subject at which they could excel:

C: When we first came to this school I had always had really high marks for maths, now I've just gone downhill.

JB: Do you know why that is?

C: I feel rushed, some areas, I don't understand, he just rushes through and I still don't understand it. (Carly, AH, Year 10, Set 1)

The experiences and attitudes of the high-ability girls in the top set at Amber Hill give some indication of the possible reasons for the gender imbalance reported at the highest levels in mathematics. Through this and other research (Boaler, Wiliam, & Brown, 2001), I have come to realize that many high set environments in England are uncomfortable places for girls because teachers tend to adapt a fast and procedural teaching approach. In a subsequent study of six typical schools in greater London, I and other researchers interviewed students in different ability groups. All the girls interviewed in the highest groups wanted to move down into lower groups, although this may have affected their examination grades because they did not like the environment of their class. Top set or high-track classes do not have to be fast, pressured environments, but many of them are. That simple fact alone may account for the underachievement of girls at the highest mathematics levels. Further evidence for this suggestion is provided by the work of Dweck (1986). Through a review of different research studies from the social-cognitive framework, Dweck has shown that maladaptive motivational patterns affect motivation and influence the quality of performance. She also showed that tendencies toward unduly low expectancies, challenge avoidance, ability attributions for failure, and debilitation under failure have been especially noted in girls, particularly "bright" girls.

Dweck asserts that one of the characteristics of "maladaptive" motivational patterns is a tendency to seek situations that lead to good performance, rather than situations that involve challenge and in which students may learn. But I would question whether such tendencies can really be described as maladaptive in many of the mathematics classrooms in which the girls are learning. In classrooms such as Amber Hill, students are rewarded for the number of correct answers they get, not for the acquisition of understanding. In such classrooms, it seems unreasonable to expect students to seek difficult and demanding situations that may not lead to correct answers, particularly when correct answers, in a mathematics classroom, have always been the only route to success. Dweck's suggestion that bright girls underachieve because of maladaptive tendencies may be seen as an example of blaming the victim (Anyon, 1981). One result of this could be that the blame is removed from the school system and focused on the reported inadequacies of girls. But the tendency to avoid situations that result in failure, taken in the context of high-pressure mathematics classrooms (such as top sets), is not at all maladaptive. In many ways, it is eminently sensible. High-pressure environments that expose students when they do not attain correct answers (Buxton, 1981) cannot foster a desire in students to seek challenging situations in which they may not succeed:

JB: Can you describe a maths lesson which you haven't enjoyed?

L: Where he was doing something about perimeters of circles and radiuses and that and he picked me out, because I didn't look interested and he was telling me all these things and I had to work it out and I just sat there, I didn't know anything, 'cause I didn't think he explained it and he made me look a fool in front of the whole class, yeah, 'cause I just couldn't speak, 'cause I didn't know what he was talking about and he goes "see me after the lesson." (Louise, AH, Year 9, Set 1)

The high-pressure environments generated within many mathematics top sets probably encourage and reinforce the tendencies Dweck notes among bright girls. It also seems reasonable that girls should become anxious (Tobias, 1978) in response to these environments, rather than reposition their goals and replace their desire for understanding with a desire for speed, as many of the boys seemed to have done:

P: Some of the stuff you do, it's just hard and some of it's really easy and you can just remember it every time, I mean sometimes you

> try and race past the hard bits and get it mostly wrong, to go onto the easy bits that you like. (Paul, AH, Year 9, Set 1)

In England, there is evidence that mathematics is taught to setted ability groups in approximately 94% of schools (OFSTED, 1996; reported in *The Guardian* 8/6/96 p. 7). The negative attitudes reported among bright girls (Dweck, 1986) and the inequities present amongst the top 5% of students (Askew & Wiliam, 1995) may derive from some of the intrinsic features of top set mathematics classrooms, rather than the personal inadequacies of girls. At Amber Hill, the top set girls were clear about the reasons for their disaffection and underachievement, and these did not relate to their own shortcomings, but to the way in which mathematics was presented to them within their fast and pressured top set classrooms.

Attribution Theory

Attribution theory (Ames, 1984; Ames, Ames, & Felker, 1977) has focused on girls' anxiety and their tendency to attribute their failure to their own perceived lack of ability. This has been used by psychologists and educationists to suggest ways in which girls should change—ways in which they should become less anxious and more confident. Anyon (1981) has described a tendency toward "blaming the victim" and this process is evident in much of the research based on attribution theory and "intervention strategies" (Mura, 1995). In such research, the responsibility for change is laid firmly at the feet of the girls. The reasons for their actions are ignored, and potential problems with mathematical epistemology and pedagogy are not considered. One of my aims in this chapter is to identify the reasons for girls' adverse reactions to school mathematics and to give voice to their concerns (see also Boaler, 1997a, 1997b, 2002b). The girls at Amber Hill talked openly about their mathematical anxiety, but they did not attribute this anxiety to their own deficiencies. They were quite clear about the reason for their anxiety, which was the system of school mathematics they had experienced.

> H: If we don't understand it, he'll shout at us, call us idiots in other words, but it's his own teaching. (Helen, AH, Year 10, Set 1)

> M: Every report he writes, he writes good ability but lacks confidence, but I know that I can do the work—in a different situation, with a different sort of work. (Maria, AH, Year 10, Set 1)

The Amber Hill girls clearly attributed their underachievement to the mathematical pedagogy and epistemology they experienced (Burton, 1995; Johnston, 1995; Mura, 1995; Willis, 1995). Although many of the girls believed they were mathematical failures and demonstrated anxiety in lessons, none of the girls related this to their own perceived inadequacies. They felt they had been disadvantaged by their school's mathematics teaching and, "in a different situation, with a different sort of work," they could have done well.

Amber Hill Summary

The girls at Amber Hill experienced an important conflict. They believed in the value of understanding, and they knew there was a need to think about work. Yet their school's approach did not always allow them to do so. When they worked at their own pace, when they worked in groups, and when they worked on open-ended projects, they felt able to gain access to understanding. Hence their preference for these approaches. The majority of the boys at Amber Hill also preferred a more open, reflective approach, but in the absence of this they seemed able to adapt to a system they did not like, but that gave them high marks. The boys were not happy, but they were able to play the game, to abandon their desire for understanding, and race through questions at a high speed. Dweck (1986) has talked about the importance of students' goals to their subsequent success and failure in cognitive performance. It was clear that the goals that the Amber Hill girls formed were almost impossible to achieve in their mathematics lessons, and the effect of this conflict on their regard for mathematics was clear.

PHOENIX PARK SCHOOL

The Phoenix Park students worked cooperatively on projects at all times: They were given the freedom to develop their own styles of working, they were encouraged to think for themselves, they discussed ideas with each other, and they worked at their own pace. In these respects, the approach at Phoenix Park matched the idealized learning environment represented by the girls at Amber Hill. Not surprisingly perhaps, gender differences were evident among the students at Phoenix Park, and these worked in favor of the girls. However, these affected a relatively small number of students, and they did not result in widespread disaffection and underachievement.

In chapter 5, I described a group of students—mainly boys—who resisted the Phoenix Park approach. These students related their low motivation to the open approach and as they, and some of the girls reported in chapter 5, they wanted more structure in their work—they wanted someone to tell them what to do; "They didn't want to find things out for themselves" (Anna, Phoenix Park, Year 10). The fact that this response was concentrated among a small group of boys suggests that it was gender-based. Martin Collins, the mathematics coordinator at Phoenix Park, believed that some of the boys lacked the maturity to take responsibility for their own learning, and there was some evidence that this was true. For example, in Year 9 interviews, some of the boys were extremely antagonistic toward the approach, but by the end of Year 10 they were considerably more positive. This may have been due to the fact that they needed time to get used to the demands of an open approach, or that they had simply become more mature by the time they reached Year 10.

The boys who appeared to be disaffected because of the Phoenix Park approach were in the minority and they demonstrated similar low motivation and bad behavior in all of their lessons (although most, but not all, of these were project based). Thus, the gender-based responses at Phoenix Park were different from those at Amber Hill. At Amber Hill, they were more consistent and widespread, and they affected girls who were both successful and motivated in other subject lessons. The disaffection of the Phoenix Park boys was global, whereas the disaffection of the Amber Hill girls was local—it related only to mathematics. Also, the girls and boys at Phoenix Park did not develop different perceptions about mathematics. Earlier in this chapter, I described a Year 9 questionnaire item in which students were asked to rank five different aspects of mathematics in terms of importance. This produced significant gender differences on three of the five mathematical features at Amber Hill and no significant differences at Phoenix Park. I also showed that there were significant differences in the responses of Amber Hill girls and boys to three statements in their Year 10 questionnaire describing different aspects of mathematics. There were no significant differences between the girls and boys on any of these questions at Phoenix Park. This is important because, at Amber Hill, the girls seemed to value aspects of mathematics teaching and learning that were not present in their school's approach. At Phoenix Park, the views of girls and boys were consistent with the approach they encountered at school.

Further indications of the gender patterns at the two schools were provided by the Year 8 questionnaire. One of the questions asked students whether they were good at mathematics in school. The responses are given in Table 9.1.

At Phoenix Park, where school mathematics was open, experiential, and discussion based, similar proportions of boys and girls reported that they were "good" at mathematics, but more of the boys said they were *bad* at mathematics. At Amber Hill, the proportion of students thinking they

TABLE 9.1
Do You Think You Are Good, OK, or Bad at the Maths You Do in School? (%)

School		*Good*	*OK*	*Bad*	*n*
AH	g	6	80	13	82
	b	32	66	1	77
PP	g	23	72	5	43
	b	22	65	13	60

were *bad* was reversed, with the girls less confident, and significantly higher proportions of boys thought they were good at mathematics. The Amber Hill data conform to stereotypical gender patterns, whereas the Phoenix Park data do not.

In their Year 8 questionnaire, students were also asked to write sentences about the aspects of lessons they liked, disliked, and would like changed. In response to these three questions, there were 88 comments from Amber Hill students about their perceived lack of understanding. The majority of these comments reflected a considerable amount of anxiety, and more than two thirds of the comments were given by girls. At Phoenix Park, there were six comments in response to these three questions that reflected anxiety about understanding, and these came from equal numbers of girls and boys.

In interviews, the Phoenix Park girls also gave different responses to the Amber Hill girls. Many more of the Phoenix Park girls reported enjoying mathematics because they worked in open, noncompetitive environments in which they could use their own ideas and think deeply about their work.

In GCSE examinations, there were significant disparities in the achievements of Amber Hill girls and boys, with 20% of the boys and 9% of the girls who were entered attaining GCSE grades A to C. At Phoenix Park, there were no significant differences in the achievements of girls and boys, with 13% of the boys and 15% of the girls attaining grades A to C. Thus, approximately equal proportions of students at both schools attained grades A to C, but these were distributed equitably at Phoenix Park and inequitably at Amber Hill.

DISCUSSION AND CONCLUSION

In concluding this chapter, I draw together a number of points that, illuminate or contradict existing theoretical standpoints consistently deployed within education and psychology relating to girls and mathematics.

1. At Amber Hill school, a large proportion of girls became disaffected by and disillusioned with their school mathematics. These girls achieved less than a similar cohort of girls at Phoenix Park and were considerably more disaffected. The girls at Amber Hill were eloquent about the reasons for their disaffection and underachievement, and these related to pace, pressure, closed approaches that did not allow them to think, and a competitive environment. Conversely, they related open work, discussion, and cooperation to understanding. Burton (1986a, 1986b, 1995) has proposed that process-based mathematical approaches raise the achievement and enjoyment of girls, but to date there has been little research evidence to support this.

2. The difference between the achievement of girls and boys at Amber Hill in relation to a traditional, closed approach appeared to relate to their adaptability to an approach they disliked. Both sets of students expressed preferences for open, discussion-oriented work, but boys adapted to the converse of this, whereas the girls generally did not. The boys tended to rush through questions to achieve speed, if not understanding. The girls would not do this; they seemed unable to suppress their desire for understanding and continued to strive toward it, which probably worked to their disadvantage.

3. Attribution theory has played an important part within psychological analyses of girls' underachievement in mathematics. Various psychologists have suggested that girls tend to attribute their lack of success to themselves, and Dweck (1986) proposes that this leads to a condition known as "learned helplessness."

Attribution theorists have tended to rely on experimental evidence to support their claims, and it is interesting to contrast this evidence with the reported experiences of girls in real classroom situations. At the end of 5 years of secondary schooling, the girls at Amber Hill were clear about the reasons for their lack of success in mathematics, and these had nothing to do with their own inadequacies. The Amber Hill girls found they were unable to improve their situation—not because they were disillusioned by their own inadequacies, but because they were powerless to change their institution's epistemological and pedagogical traditions.

4. Dweck (1986) analyzed the negative reactions of girls to school mathematics and described their responses as "maladaptive" (p. 1040). I have argued that the girls' responses should be considered in relation to their goals in mathematics; if their goals relate to understanding, which they clearly do, their responses are far from maladaptive. We are now emerging from a period in educational research in which quantitative measures were relied upon and few sought the opinions of girls. In 1986, Burton argued that intervention strategies designed to make girls more confident and/or successful would be ineffective if they did not attempt

to locate and understand the nature of girls' "problems" from a broad perspective (Burton, 1986b). Few researchers consulted the girls or listened to their concerns before labeling them as "anxious" and sending them on programs to become more confident. Yet it seems that the Amber Hill girls' responses to school mathematics make a lot of sense. Indeed, their proposals for the improvement of school mathematics are markedly similar to those offered by experienced mathematics educators. They want to be able to understand mathematics, and they will not accept a system that encourages rote learning of methods and procedures that mean little or nothing to them.

5. Previous research that has considered the links between sex and learning styles has reported small or negligible effect sizes. This has led educationists to dismiss any possible differences between girls and boys partly because it would be dangerous to form expectations on the basis of presumed learning styles (Adey, Fairbrother, Johnson, & Jones, 1995). However, it seems equally dangerous to ignore sex-based preferences for learning styles when the teaching approaches offered to school students are clearly biased toward one group of students. Mathematics, as it is currently and widely taught, is not equally accessible to girls and boys, and this appears to relate to preferences of pedagogy. Many of the psychological studies that have reported negligible learning style differences between girls and boys (see e.g., Riding & Douglas, 1993) have done so by reducing learning preferences to small measurable concepts related, for example, to a verbal versus imagery approach or a holist versus analyst approach. These are then assessed through closed questionnaires administered in experimental settings. One of the indications of this research study was that the preferences of girls for open, reflective, and discursive approaches that allowed a connected type of thinking (Becker, 1995; Gilligan, 1982) would not be easily identifiable through experimental tests for learning styles. The Amber Hill students' preferences were related to pedagogy and the breadth and depth of the mathematics they met. Such preferences emerged in response to the practices of their mathematics classrooms and were more a part of their school environments than the girls themselves.

6. The disparity between preferred modes of working and school mathematics practice was most acute for the highest ability girls at Amber Hill. In recent years, girls' performance in mathematics has improved dramatically in relation to boys (Elwood, Hayden, Mason, Stobart, & White, 1992), but important differences still persist among the top 5% of students. The disaffection and underachievement, which was common among the highest ability girls at Amber Hill, derived partly from the increased pressure and speed associated with their teaching environments as well as the increased awareness of the girls of the inadequacy of an approach that denied them access to understanding. These girls, more than others, wanted

to understand their mathematics. Consequently, these girls, more than others, become anxious and underachieved when they were denied the opportunity to do so.

I began this chapter with a quote from Johnston (1995), which suggested that it may be time to listen to the girls who complain about the nature of school mathematics. I have attempted, in this chapter, to show the importance of giving voice to girls' concerns because what they are saying appears to make a lot of sense. However, it is important not to lay the blame for their disaffection on mathematics per se because the fault lies not with the intrinsic nature of mathematics, but with school mathematics as it is commonly constructed. Rogers and Kaiser (1995) talk about the need to move away from a paradigm that has blamed girls for the pedagogical and institutional inadequacies of the school system and move toward a new form of school mathematics. At Phoenix Park, the teachers were quite radical in their reconstruction of school mathematics, and this seemed to produce an alleviation or even eradication of mathematical anxiety and underachievement among girls. More important, they achieved this by changing the mathematics pedagogy and epistemology, not the girls.

One of my aims in writing this chapter has been to oppose a discourse that positions girls as incapable. Such a discourse is pervasive even among educational researchers, with some analyses in mathematics education suggesting that girls *cannot* rather than *do not* achieve (see also Boaler, 2002b). The results of this study show that beliefs, confidence, and mathematics achievement vary according to teaching environments and are not a feature of being female. I have suggested that some girls have different preferences from some boys, but I do not want to suggest that this is due to the students' gender. Girls and boys, as well as students from different cultural groups (Gutierrez, 1996, 1999), construct their understandings, beliefs, and preferences in relation to their working environments, and it seems that some environments coproduce unproductive gender responses and some do not. Although the underachievement of particular groups of students—such as the girls at Amber Hill—should prompt investigation, such investigations should aim to understand the relationship between the students' responses and the environments that produce them. Not all teaching approaches are inequitable, and female underachievement should never be considered a "corollary of being female" (Rogers & Kaiser, 1995).

10

Ability Grouping, Equity, and Survival of the Quickest

Whether schools should group students by perceived ability is one of the most contentious issues in education. Some research gives evidence of the impact of ability grouping, but there have been few opportunities to monitor the experiences and attainment of a matched group of students as they move through different systems of grouping, as was presented in this study. In this chapter, I therefore aim to unpack the influence of the different grouping practices employed at Amber Hill and Phoenix Park for attainment generally and for the achievement of equity. This shows some surprising results.

Before I begin this analysis of ability grouping, I should point out that ability grouping practices in England and the United States share some similarities and some differences. In England, students are often placed into ability groups called *sets* at a young age (and there is a governmental push to make it younger), and they are then taught accordingly, with high sets being prepared for high-examination grades and low sets being prepared for low grades. Such grouping practices occur at subject specific levels—with mathematics departments employing ability grouping more than any other subject department.

At Amber Hill, the students were divided into ability groups at age 13 (a relatively late age for England); from that time, their potential attainment at age 16 was narrowly restricted. The students were divided into eight groups, and this fine level of setting is not unusual. Some schools divide students into 16 or 20 ability groups if there are sufficient students in a year cohort. One of the ironies of this system is that students are rarely told the implications of the group within which they are placed. Sometimes they are not even told the level of the group they are in, and administration may go to great lengths to hide the grouping, naming the groups after colors instead of numbers. The reason for this secrecy is that teachers do not want to demotivate students. Such motives are understandable; in my first day of my first teaching position, I was faced with a *bottom* group of students (Set 4 of 4), and their opening question to me after I introduced myself was, "Why should we bother?" (We subsequently desetted the mathematics classes.) The subterfuge continues when teachers teach lower level content to lower groups at a slower pace—the students are always successful, and they often become convinced that they are high-attaining students, not realizing they are being taught particularly low-level work. When students are told in Year 10 that the highest examination grade they can possibly attain is a D or an E, they often feel genuinely cheated. Yet teachers feel that this ultimate demotivation is preferable to demotivation at an earlier age.

In the United States, the implications of tracking are often easier to discern (e.g., some mathematics classes are identified as *college prep* and some are not). Nonetheless, the two systems can work in similar ways (e.g., when U.S. students are not informed by their guidance counselors that taking only 2 years of high school mathematics will make them ineligible for admission to many postsecondary institutions). However, there are also important similarities between ability grouping in the United States and England. The central one is that they have both evolved in response to the question of how best to maximize the potential of a range of students with varied interest, attainment, and apparent potential. Many recognize that setting and tracking sort by demographics—particularly ethnicity and socioeconomic status (SES; Oakes, 1985)—as much as by attainment, and they recognize that it can restrict attainment for students, but cannot see how "bright" students may be stretched or "weak" students supported in a mixed-ability system. This extremely important question is centrally addressed in this chapter through a consideration of the ability grouping practices at the two schools and analysis of the relationships between ability grouping and achievement for the students at the two schools.

ABILITY GROUPING AT AMBER HILL

The Amber Hill mathematics department was an interesting place to consider the impact of ability grouping because the students were taught in mixed-ability groups for 2 years when they were in Grades 6 and 7 and then in sets for mathematics in Grades 8, 9, and 10. These differences gave the students extremely interesting insights into setted and mixed-ability approaches. It became clear as I was researching the students' experiences that their responses to their mathematics teaching were hugely impacted by the ability grouping practice they experienced. I now discuss in turn the particular aspects of ability grouping that impacted students the most, starting with the one that seemed to have the greatest effect on the largest number of students.

Working at a Fixed Pace

Probably the main reason that teachers place students into ability groups in mathematics is so that they can reduce the spread of ability within the class, enabling them to teach mathematical methods and procedures to the entire group as a unit.

> It's good [setting] because you're putting similar abilities together. I mean it's easier to pitch your lesson, to pitch the work at them, to teach them all together, you know, from the front, as a class. (Edward Losely, mathematics teacher, Amber Hill)

There is evidence that the way in which teachers proceed in setted lessons is by teaching toward a reference group of students (Dahllöf, 1971). Teachers generally pitch their lessons at the middle of the group on the basis that faster or slower students adjust to the speed at which lessons are delivered. At Amber Hill, students worked through individualized booklets at their own pace in Grades 6 and 7. This is a common approach used in England—when students finish their current booklet, they get the next one. The teacher helps individual students, and many students enjoy the opportunity to decide the pace of their own work. When the Amber Hill students changed at the end of Year 7 from working at their own pace to working at a fixed pace, many students became disaffected and their attainment started to decline. The view that working at a fixed pace diminished understanding was prevalent both among students who found their new lessons too fast and students who found them too slow. Yet these were not unusual or extreme students; almost all of the students seemed to find some lessons, or some parts of lessons, either too fast or too slow:

> C: I felt like I was learning—you feel you was learning more, 'cause the teacher would help you—if you went up to him and showed him the book he would help you and I felt I learned more in the first and second year, but in the fourth and fifth year it's more slow and like if you finish first you have to wait for the others, or if you're behind you have to work fast because everyone else is finished.

M: And that's why I don't like maths any more 'cause I can't go at my own pace. (Chris & Marco, AH, Year 10, Set 4)

The pace at which students felt comfortable working seemed to be determined by a wide range of factors. These included the difficulty of individual topics, students' own prior experience, individual preferences, and, of course, their feelings on that day.

The fact that Amber Hill used ability grouping did not mean that the teachers *had* to teach students as a group at a fixed pace, but for many teachers in England the only reason for establishing ability groups is to enable teaching the same content to whole classes. When teachers teach to mixed-ability groups, they find various ways to differentiate material—some using open-ended materials (the Phoenix Park approach) that enable students to take work in different directions and levels. Some differentiate by preparing different work for different students. When teachers place students into ability groups, however, they often forget that students are still at different places in their mathematics learning, with different strengths and needs; they assume that the students are now a homogeneous group and differentiation is not required. This assumption, and the teaching practices associated with it, caused a number of problems for the Amber Hill students.

The students' second major complaint about setting was also related to class teaching, but it extended beyond this. A major concern of significant numbers of students interviewed was the pressure they felt was created by the existence and form of their setted environments.

Pressure and Anxiety

Many of the Amber Hill students, particularly girls, were anxious about mathematics, and the students linked their anxiety to the pressure created by setted classes. Some of this pressure derived from the need to work at a pace set by the teacher:

H: I don't mind maths but when he goes ahead and you're left behind, that's when I start dreading going to maths lessons. (Helen, AH, Year 10, Set 1)

K: I mean she's rushing through and she's going "We've got to finish this chapter by today" but I'm still on C4 [a textbook exercise] and I don't know what the hell she's chatting about and I haven't done any of it, 'cause I don't know it, she hasn't explained it properly she just says "take this off, take that off" and she puts the answers up and like—what?, I don't know what she's doing. (Karen, AH, Year 10, Set 3)

Another aspect of the students' anxiety related to a more reflective pressure. The creation of groups intended to be homogeneous in ability caused many students to feel that they were constantly being judged alongside their peers:

L: I preferred it when we were in our tutor groups.

JB: Why?

L: 'Cause you don't worry so much and feel under so much pressure then, 'cause now you've got people of the same standard as you and they can do the same stuff and sometimes they can do it and you can't and you think oh I should do that and then you can't . . . but if you're in your tutor group (mixed ability) you're all a different status . . . it's different. (Lindsey, AH, Year 10, Set 4)

One of the reasons commonly given for the formation of setted groups is that the competition created by setted classes helps raise achievement. For some students, this was probably true:

B: You have to keep up and it actually, in a way it motivates you, you think if I don't do this then I'll get behind in the class and get dropped down a set. (Gary, AH, Year 10, Set 3)

However, of the 24 students interviewed in Year 10, only 1, Gary, gave any indication that the competition and pressure created by the setted environments enhanced motivation or learning. At Amber Hill, setting was a high-profile concept, and the students were frequently reminded of the set to which they belonged. This served as a constant standard against which they were judged, and the students gave many indications that this continual pressure was not conducive to their learning.

Top Set Experiences

Undoubtedly, the most intense pressure in mathematics lessons was experienced by students in the top set; at Amber Hill, placement in the top set appeared to have serious negative consequences for the learning and achievement of some students (Boaler, 1997a). Most, but not all, of the students who were negatively affected were girls, and I described some of the ways in which these girls were disadvantaged by their top set experiences in the last chapter. The top set of my case study year group was taught by the head of mathematics, Tim Langdon, who was ambivalent about setting:

> A lot of people are not prepared to take on board mixed-ability and if I'm speaking as a head of department, I'm obviously trying to look to maximize what people I've got in my department in front of me, so if we move the question on to what I can see, I can see a whole bunch of people who are happy with sets, sets by ability and we'll stick with that and look for making them feel comfortable so they're prepared to give me as much as possible. If, from my own point of view, yes I would like some mix of ability within a group because I still feel there's some trickle down effect and still more positive effect within a group with a spread of ability. (Tim Langdon)

Despite Tim's ambivalence toward the setting process, the environment within his own top set group embodied many of the features that characterize top set mathematics groups—particularly rapidly paced lessons, competition among students, and pressure to succeed. In my observations of Tim's top set class, I was often surprised by the pace of the lessons compared with the lessons he taught to other sets. All of the Amber Hill teachers taught lessons at a pace I would regard as reasonably fast, but the top set lessons were distinct. The identification of students as *top set* seemed to trigger a whole variety of heightened expectations for the teachers about the students' learning capabilities. It was almost as if the teachers believed they were dealing with completely different students—those who did not experience problems, who understood the meaning of examples flashed up on the board for a few seconds, and who could rush through questions in a few moments, deriving real meaning from them as they did so. In the following extract, Lorna and Jackie, two of the top set girls, described their lessons to me:

L: So he'll go through, like notes on the board and go through questions and ask us questions and then . . .

J: Leave us to it.

L: But sometimes, when we've got to get a chapter finished, we go through it *so fast* and sometimes we don't know where we're at, like what we're supposed to have done, what we're, you know, what's coming up.

J: It feels like the teacher's skipping things but he's not, it's just that we've got to go through it so fast.

L: Yeah and sometimes you forget what you've done don't you?

J: Yeah.

L: Like you've just taken one thing in and then you've got to switch to the next chapter or the next piece, it's confusing.

J: Yeah you get really confused. (Jackie and Louise, AH, Year 9, Set 1, with students' own emphasis)

In interviews, the top set students were distinct from students in other groups by virtue of their discourse—in particular, their constant reference to the pace of lessons using words like *speed, zoom, fast,* and *whizz*:

H: All we've been doing for weeks is practicing exam papers, but even that, you just zoom through it, you can't take your own time to do it, and then, it's when you come to the lesson, he's just zooming through it, and still you can't get, you don't understand it properly. (Helen, Year 10, Set 1)

In order to monitor whether the features of Tim's top set teaching were common to other top set groups taught by other teachers, I observed lessons from other year groups. This showed that many of the same features, particularly the speed, pressure, and competition, were emphasized in other Set 1 classes. Indeed, the top set lessons taught by different teachers seemed to have more in common with each other than with lessons taught by the same teachers to different ability groups. Hilary Neville usually taught Set 3 or 4 classes, but she had one top set group in Year 6. During my observations of this class, I was struck by the similarity between these lessons and other top set lessons with different teachers and year groups. Hilary seemed to change into a different teacher for these lessons; she treated the students differently, and her explanations were very fast. The top set lessons in all the year groups were taught with an air of urgency—almost as if the status of the students meant that the lessons had a completely different agenda than lessons taught to students in other groups. The students also reported that the teachers had different expectations of them because they were in the top set:

JB: Can you tell me about being in set 1?
H: They expect you to know more.
M: Yeah, they expect too much, it's like 'oh you should know this. . . .
H: You should know that.
M: You're the top set.
M: And he goes *fast*, like we'll be on one chapter one lesson and the next lesson it'll be "we've done enough of that, go onto the next one."
H: Yeah and it's, Oh my God it's, I mean I know it's the same in every lesson, but they, like set you so much work in maths and they ex-

pect you to definitely have it in by next time, and it's . . . all subjects do that, but, in maths, it's different.

M: It's tough.

H: Yeah, it's tough. (Helen and Maria, AH, Year 10, Set 1)

In questionnaires given to the students when they were in Year 8, I asked them to describe themselves as *good, OK,* or *bad* at the mathematics they did in school. No girls and only two boys in Set 1 described themselves as *good* at mathematics. In their Year 9 questionnaires, the students were asked whether they enjoyed mathematics lessons *always, sometimes,* or *never.* The Set 1 students were the most negative group in the year, with the smallest proportion of students' responding *always* (0%) and the greatest proportion of *never* (27% – six girls and two boys). Sets 1 and 2 between them contributed over two thirds of the *never* ratings from Sets 1 to 8 (n = 163).

A number of different research studies have linked mathematical enjoyment with mathematical ability or competence. Understandably, students who are good at mathematics tend to enjoy it, whereas students who experience successive failure in mathematics tend to dislike it. At Amber Hill, the top two sets were made up of students who, at one time, were doing well in mathematics. Despite this, the students liked mathematics less than other students and had less confidence in their own ability to do mathematics. For these students, something had clearly gone wrong. During my 3 years of work at Amber Hill, I realized that the negativity of students in Set 1 was caused by features of the class induced by ability grouping. This derived from a number of sources. First, 10 of the 12 students interviewed from Set 1 expressed a clear preference for mathematics lessons in Years 6 and 7 when they worked in mixed-ability classes using an individualized approach:

J: 'Cause you learned a lot more [in mixed-ability groups] and you could recap everything which you didn't understand and spend more time on it, but now you've just got to try and whizz and do your best. (Jackie, AH, Year 9, Set 1)

Second, in questionnaires given to all of the Year 9 students, 17 of the 30 top set students gave comments similar to the ones below:

The teacher rushes through methods faster than most pupils can cope.

The lesson is difficult, and we work at such a fast pace that I find it hard to keep up.

I dislike basically everything. The methods of teaching are too fast and confusing.

Third, when in their Year 8 questionnaire the students were asked to name their best ever mathematics lesson, all the students who described a mathematics lesson (n = 17) chose their coursework projects. Nine other students did not give an answer, and four students chose a lesson when a policewoman came in to give a talk about weapons. The 17 students who prioritized their coursework lessons over all others said they did so because they valued the opportunity to work at their own pace, to find things out for themselves, and to experience a less confrontational style of learning. None of these features has to be associated with ability grouping; it is possible for a teacher to have a high set class and give differentiated work, treat students as individuals, and avoid competition and pressure. But at Amber Hill, the teachers incorporated these features into their classes when the groups became setted. Furthermore, this seems to be a common experience. In a follow-up study of six schools, I found the same characteristics of high-set groups and the same levels of student disaffection among students in high groups (Boaler, Wiliam, & Brown, 2001).

The Set 1 students were a group of committed and able students who should have been enjoying and succeeding at mathematics. Instead, their comments suggest considerable disaffection particularly because of the speed of lessons and the pressure they experienced. This story of negativity and anxiety was repeated in different top set mathematics groups across Amber Hill school and is a story repeated in many top set or high-track mathematics classrooms for small, but significant, groups of students. When my case study group was in Year 9, I gave a questionnaire to students in Years 8, 9, and 10 (n = 420). The Set 1 students across the three year groups responded differently from other students on this questionnaire. For example, Set 1 students comprised 26% of the students who said that they never enjoyed mathematics, 38% of the students who described lessons as fast, and 27% of the students who said they were always anxious in lessons, when Set 1 students made up only 19% of the cohort. On all of these questions, the views of the Set 1 students, taught by different teachers, were consistent across the three year groups.

JB: Can you think of some good and bad things about being in set 1?

L: I can think of the bad things.

C: I agree.

JB: OK, what are the bad things?

L: You're expected to know everything, even if you're not sure about things.

C: You're pushed too hard.

L: He expects you to work all the time at a high level.

C: It makes me do less work, they expect too much of me and I can't give it so I just give up. (Carly and Lorna, AH, Year 10, Set 1)

The students indicated that the nature of their top set environment had diminished their understanding of mathematics. This idea was validated by a number of the different assessments reported so far. Two class groups at Amber Hill were the focus of the long-term learning study. One was a top set Year 8 group, the other a Set 4 Year 9 group; both were taught by the same teacher—Edward Losely. In a comparison of the instances of positive learning, (where students learned something and remembered it), with instances when they learned something but then forgot it, the Year 8 Set 1 group did significantly worse than the mixed-ability Year 8 group at Phoenix Park and the Year 9 Set 4 group at Amber Hill. Indeed, in this top set group, 10 out of 22 students (45% of the group) attained lower scores in the delayed posttest than they did in the pretest taken before the work was introduced to them. This compared with two students from the Year 8 Phoenix Park group and no students in the Year 9 Set 4 group at Amber Hill. Although that research was of a small scale, it showed quite clearly that the learning of the Year 8 top set students, on the particular piece of work assessed, was extremely ineffective and, for almost half of the group, it may even have been detrimental. Nothing about this work made it distinct from any other piece of work the students did, and in my observations of their lessons the students were motivated and worked hard. Edward taught them methods at the usual pace for the class; the students watched, listened, and then practiced the methods as was normal for the school.

The students in my top set case study group also attained the lowest grades, of Sets 1 to 4 , on both aspects of the applied architectural activity and the area question in the flat design activity. The students in Set 1 seemed to have particular difficulty working out what they should do within these assessments possibly because they had learned methods at a faster pace than other students and were particularly prone to making cue-based decisions in an attempt to get by in lessons. Further indication of the difficulties experienced by top set students was provided by the conceptual and procedural results reported in chapter 6. These results show that students who took the higher level examination paper in the top set at Amber Hill were less able than other students to answer conceptual questions, and this contrasted strongly with the most able students at Phoenix Park. In three different assessments, the top set students showed that their experiences of mathematics class may have disabled them in a variety of situations. The negative features they highlighted—pressure, fast-paced lessons, and assumptions of homogeneity—do not have to be associated with ability grouping and they may be present in any mathematics class, but they all combined to detract from the students' opportunities to learn.

The third major complaint of Amber Hill students was particularly prevalent among students outside Set 1; it related to the way in which setting limited their potential opportunities and achievements.

Restricted Opportunities

In interviews, many of the Amber Hill students expressed clear feelings of anger and disappointment about what they felt to be unfair restrictions on their potential mathematical achievement. The students, from a variety of sets and ability ranges, cared about their achievement. They wanted to do well, and they were prepared to put effort into their work, but many felt they had been cheated by the setting system:

L: The thing I don't like about maths is . . . I know because we're in set 4 you can only get a D.
S: Yeah you can't get any higher than a D.
L: So you don't do as much.
S: Yes you could work really hard and all you can get is a D and you think, well what's the point of working for a D? (Lindsey and Sacha, AH, Year 10, Set 4)

A: I'm in set 3 and the highest grade I can get is a C . . . it's silly because you can't, maybe I wanted to do A-level, 'cause maths is so useful as an A-level, but I can't because . . . I can get a C if I really push it, but what's the point? (Alan, AH, Year 10, Set 3)

A number of the students explicitly linked the restrictions imposed by the set they were in to their own disaffection and underachievement. They reported that they simply could not see any point in working in mathematics for the grades available to them:

JB: How would you change maths lessons? If you could do it any way you wanted what would you do?
C: Well work at your own pace and different books.
JB: How would working at your own pace help?
M: Well it would encourage people more wouldn't it?, they'd know they're going for an A wouldn't they? like what's the point of me and Chris working for a D? Why are we gonna work for a D?
C: I'm not saying it's not good a D, but . . .

M: It's *not* good, it's crap, they said to us if we get 100% in our maths we're gonna get a D, well what's the point? (Chris & Marco, AH, Year 10, Set 4, with students' own emphasis)

These extracts raise questions about the accuracy of the students' assessments of their own potential, but in many ways the degree of realism in the students' statements is irrelevant. For what the students clearly highlight is the disaffection they felt because of the limits imposed on their attainment. The students may have been unrealistic, but the disaffection they experienced because of their restricted attainment was real. Few researchers have interviewed students in the United States who do not get placed into the algebra class with their cohort, who get held back in classes, and who never reach precalculus by the end of high school, but it seems likely that similar views would be expressed.

S: We're more to the bottom set so we're not expected to enjoy it.

JB: Why not?

S: I'm not putting, I'm not saying 'cause we're in the lower set we're not expected to enjoy it . . . it's just . . . you're looking at a grade E and then you put work in towards that . . . you're gonna get an E and there's nothing you can do about it and you feel like . . . what's the point in trying, you know? what's the difference between an E and a U?

JB: How did you feel about maths before you were put into sets?

K & S: Better. (Keith & Simon, AH, Year 10, Set 7)

These feelings of despondency were reported from students in Set 3 downward at Amber Hill, and many of the students suggested that the limits placed on their attainment had caused them to give up on mathematics. The students believed that they had been restricted, unfairly and harmfully, by their placement into sets. The fourth and final response that prevailed among students primarily affected the students in low sets, and this related to the way in which the sets were chosen.

Setting Decisions

Many of the students interviewed did not feel that the set they had been put into was a fair reflection of their ability:

S: I was alright in the first year, but like me and my teacher had a few problems, we didn't get on, that's why I think it's really better to work really hard in the first years, 'cause that's when you've got a chance to prove a point, you know, that you're good and then in

> the second year you'll end up in a good set and from then on you can work. But me in the first year, I got dumped straight into the bottom set. And I was like huh? what's going on?, you know? And they didn't teach me anything there and I was trying hard to get myself up, but I couldn't, 'cause once you're in the bottom it's hard to get up in maths. That's another bad thing about it, and other people now, there's people now in like higher sets man and they just know nothing, they know nothing. (Simon, AH, Year 10, Set 7)

Some of the students, particularly the boys, felt the set they were in reflected their behavior more than their ability:

> M: Yes but they're knocking us down on our behavior, like I got knocked down from second set to bottom set and now, because they've knocked me down, they've thrown me out of my exams and I know for a fact that I could've got in the top A, B, or C. (Michael, AH, Year 10, Set 7)

Tomlinson (1987) provides evidence that students' behavior can influence the groups into which they are placed, and some of the Amber Hill students were convinced that their behavior, rather than their ability, had determined their mathematics set, which in turn, had partly determined their examination grade.

Amber Hill Summary

The Amber Hill students were coherent in their views about ability grouping. The 24 students interviewed in Year 10 were in general agreement about the disadvantages they perceived, and all but 1 of the students interviewed expressed strong preferences for mixed-ability teaching. This was because, for many of the students, setting meant one or more of:

- a lack of understanding when the pace of lessons was too fast;
- boredom when the pace of lessons was too slow;
- anxiety created by the competition and pressure of setted environments;
- disaffection related to the restricted opportunities they faced; and
- perceived discrimination in setting decisions.

It was also clear from the students that setting did not have a single influence that affected all students in the same way. Some students were probably advantaged by setted lessons, but others had been negatively affected by processes of setting. In almost all cases, the disadvantages students reported concerned their learning of mathematics and their subsequent achievement. Nevertheless, some students also experienced other negative repercussions:

K: You walk around the school and you get people in the top set and you get people in our set and if you walk round the school and you're talking about maths, they put you down because you're not in that set, it's like. . . .

S: They're dissing you and that. *[showing disrespect]*

K: They're saying you haven't got the ability they've got. (Keith & Simon, AH, Year 10, Set 7)

Despite the labeling associated with setting, the major concern for the majority of students interviewed was the consequences setting might have for their achievement. In the next section, I consider the Phoenix Park students' experience, before presenting various forms of data that show the way in which the students' achievement was affected by their placement in sets.

THE MIXED-ABILITY EXPERIENCE AT PHOENIX PARK

I have written about the Amber Hill students' experiences of ability grouping in generally negative terms, reflecting the students' issues and concerns they communicated to me. At Phoenix Park, I received no negative data about grouping, which may partly reflect the fact that the Amber Hill students had something to react *against*. They had been working in mixed-ability groups in Years 6 and 7 using a differentiated, individualized approach to teaching and learning; then they were moved to setted groups with concomitant changes in the teaching environments they experienced. At Phoenix Park, the students had worked in mixed-ability groups throughout high school, and nothing about that experience seemed to suggest for them that anything should be any different.

Some students did complain to me about being in the same group as less motivated, more disruptive students, but it is unlikely that setting would have changed this because the disruptive students probably would have been distributed throughout the setted groups. In the absence of data highlighting this aspect of the Phoenix Park students' experience, I shall take the time to ask two important questions of the Phoenix Park approach: How did teachers make sure that the able students were being stretched sufficiently, and how did they make sure that the less able students received sufficient support? The achievement data demonstrate that the Phoenix Park students attained higher grades than the national average on the national examination despite being from working-class homes and experiencing an approach completely dissimilar to the examination. Therefore, it seems worth spending a little time on the particular grouping practices they used to achieve this.

The activities used by the Phoenix Park teachers were all chosen to give access to students at multiple levels and enable different mathematical investigations and explorations. Sometimes the teachers planned different paths through a problem and steered students toward those. Occasionally they offered different activities to different students. As students worked through their various activities and investigations, the teachers would help individual students, making sure that all students were taking the activities in sufficiently demanding directions and supporting students who needed more support. But the teachers never told the students what to do, and they did not subscribe to the common belief that lower attaining students needed more structure. They merely asked different questions of the students to help them make the connections they needed to make. When the students worked on the 36-fences problem, the teachers encouraged some of the students to use trigonometry, teaching them trigonometric ratios to help them answer problems they had posed; other students investigated the areas of different shapes and drew conclusions about the relationships between side length and area, but did not learn trigonometry at that time. The teachers differentiated the students' experiences through a combination of the activities they chose for the students and the help and directions they gave them.

The mixed-ability approach at Phoenix Park was most advantageous for the highest and lowest attaining students in the school when compared with similar students at Amber Hill. At age 13, the students who were attaining at the lowest level in the grade were put into low sets at Amber Hill and taught low-level work. Many of them gave up on mathematics as a result. At Phoenix Park, the students were given activities to work on alongside the higher attaining students, and they were constantly encouraged to think about mathematics and learn. One of the girls who was attaining at the lowest level in the year group when she entered Phoenix Park, and would have been placed into a low set if there had been any, worked hard and attained a GCSE grade B. This is an extremely good grade that is attained by the top 10% of students nationally. The most able students at Phoenix Park encountered many opportunities to pursue high-level investigations, which contributed to the fact that there were many more of the highest GCSE grades attained at Phoenix Park than at Amber Hill.

A number of conclusions may be drawn about the opportunities students received at the two schools and the impact these had on the students' learning of mathematics. Before doing so, it seems important to carefully consider the patterns that may be discerned from the achievement data at the two schools and the ways these data relate to the ability grouping practices of the two schools.

ABILITY GROUPING AND ACHIEVEMENT

The Amber Hill students' different responses to setting, given in interviews, indicate that the success or failure of a student in a setted group related to his or her preferred learning style and responses to competition, pressure, and opportunity (or lack of it). Various quantitative indicators add support to the idea that success was strongly related to factors other than ability. For example, at Amber Hill, there was a large disparity between the attainment of students when they entered setted lessons and their success in GCSE examinations at the end.

This may be demonstrated through a consideration of the students' scores on their NFER tests at the end of Year 7 and their scores on their GCSE examinations at the end of Year 9. This information is provided for both of the schools, providing an insight into the different implications of setted and mixed-ability teaching for students' achievement.

At Amber Hill, a high correlation would be expected between NFER results at the end of Year 7 and eventual achievement because the students were setted largely on the basis of their NFER results. Once inside their sets, the range of their attainment was severely restricted. At Phoenix Park, a smaller correlation would be expected because, prior to their NFER tests, the students had attended fairly traditional middle schools; at Phoenix Park, they experienced considerable freedom to work if and when they wanted to in lessons. This, combined with the openness of the school's teaching approach, may have meant that some students would not perform at the end of Year 10 as would be expected from their performance at the end of Year 7. A comparison of performance, before and after setting and mixed-ability teaching, at the two schools is shown in Figs. 10.1 and 10.2.

These scattergraphs display an interesting phenomenon: At Amber Hill, there was a relatively weak relationship between the students' attainment in Year 7 and their eventual success after 3 years of working in setted lessons, demonstrated by a correlation of 0.48. This meant that some students did well, although indications in Year 7 were that they were not particularly able and some students did badly despite being high achievers at

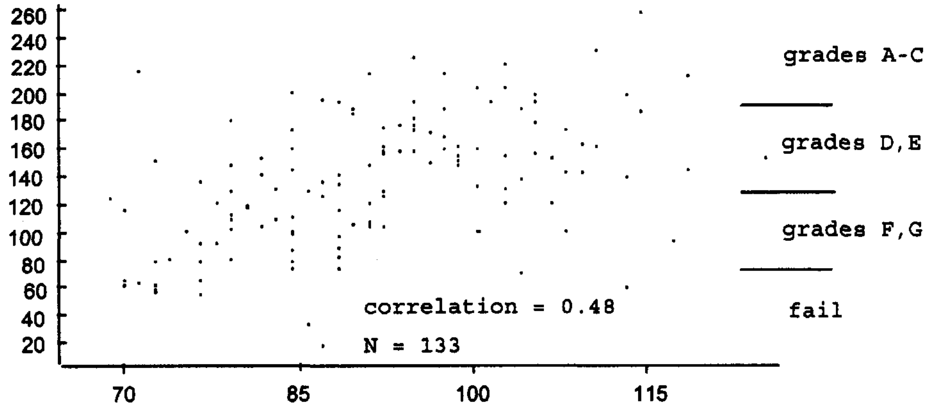

FIG. 10.1. Relationship between GCSE mark and NFER entry scores at Amber Hill.

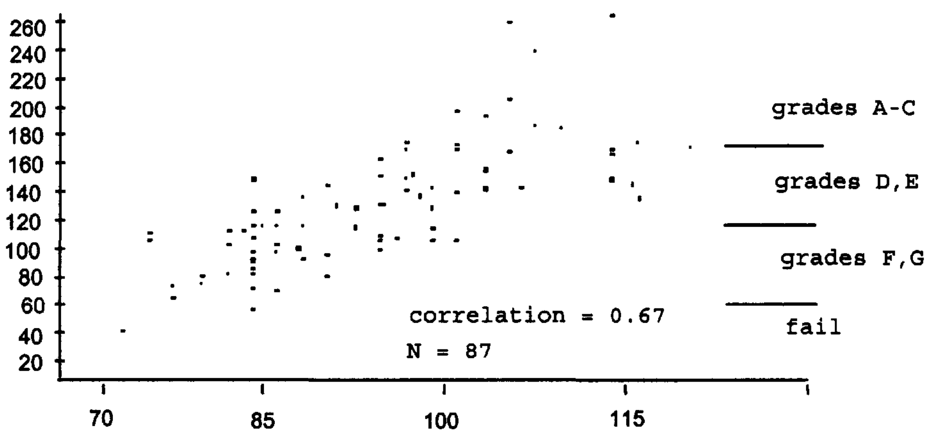

FIG. 10.2. Relationship between GCSE grade and NFER entry scores at Phoenix Park.

the end of Year 7. At Phoenix Park, where students were taught in mixed-ability groups and given considerably more freedom, there was a significantly higher correlation of 0.67 between initial and eventual attainment. These results support the idea that, once inside a setted group, a number of factors that are relatively independent of initial attainment influence student's success.

A second interesting phenomenon was revealed at Amber Hill through a consideration of the relationship between social class and the set into which students were placed. This relationship was examined at both schools because the students were put into setted examination groups at

Phoenix Park toward the end of Year 10. Partial correlations from the two schools enable consideration of the impact of ability (measured via NFER tests) and social class on the sets students were given. These showed that at Amber Hill there was a significant correlation between the social class of students and the set they were placed into ($r = 0.25$) after controlling for ability, with students of a low social class being more likely to appear in a low set. A similar analysis of partial correlations at Phoenix Park showed that there was a small, but significant, *negative* correlation between social class and examination group ($r = -0.15$) after controlling for ability. This showed that at the end of their mixed-ability teaching experiences, there was a small tendency for students of a lower social class to be placed into a higher examination group at Phoenix Park than middle-class students of similar initial attainment.

Further insight into the possibility of class bias is demonstrated by locating individuals at the two schools who achieved more or less than would have been expected from their initial entry scores. At Amber Hill, approximately 20% of the students ($n = 23$) could be described as *outliers* on the scattergraph. The 23 most extreme outliers on the graph were made up of 8 overachievers and 15 underachievers. Closer examination of these students gives the following sex and class profiles (see Tables 10.1 and 10.2). These tables show that, among the overachievers, three quarters of

TABLE 10.1
Amber Hill Students Achieving Above Expectation

	Social Class					
	Middle Class			*Working Class*		
Gender	*1*	*2*	*3*	*4*	*5*	*6*
Girls			1			1
Boys		4	1	1		

Note. Social class categories based on OPCS classification.

TABLE 10.2
Amber Hill Students Achieving Below Expectation

	Social Class					
	Middle Class			*Working Class*		
Gender	*1*	*2*	*3*	*4*	*5*	*6*
Girls			2	4	1	2
Boys	1			5		

Note. Social class categories based on OPCS classification.

TABLE 10.3
Phoenix Park Students Achieving Above Expectation

	Social Class					
	Middle Class			Working Class		
Gender	1	2	3	4	5	6
Girls					1	
Boys		1	1	4		

TABLE 10.4
Phoenix Park Students Achieving Below Expectation

	Social Class					
	Middle Class			Working Class		
Gender	1	2	3	4	5	6
Girls		1	2	1		
Boys		1	1	2	1	2

the students were middle class and mainly boys. In contrast, only one fifth of the underachievers were middle class. These outliers represent only a small proportion of the students at Amber Hill, but they show quite clearly that those students who did better than would be expected from their initial ability scores tended to be middle-class boys, whereas those who did worse tended to be working-class students (of either sex). This is interesting to contrast with the most extreme 20% of Phoenix Park students (n = 18). These students did not under- or overachieve to the same extent as the Amber Hill students, as can be seen from the scattergraphs. However, the students who were nearest to the edges of the graph did not reveal any class polarization in achievement at Phoenix Park (see Tables 10.3 and 10.4). These tables show that less than a third of the overachievers at Phoenix Park were middle class. The underachievers were made up of similar proportions of middle- and working-class students.

What these results indicate is that, at Amber Hill, the disparity between initial mathematical capability and eventual achievement shown on the scattergraph is partly created by a small number of middle-class students who achieved more than would be expected and a relatively large number of working-class students who achieved less than would be expected given their attainment on entry to the school. Similar evidence of class polarization is not apparent at Phoenix Park. This result enables social class to be added to the list of factors that appeared to influence achievement in setted lessons. It also reestablishes the notion that success in a setted environment is dependent partly on factors other than mathematics work. The influence of class bias over setting decisions is well documented (Ball, 1981; Oakes, 1985; Tomlinson, 1987) and some of the students gave some indications in interviews about the way that this process may have taken effect. In the following extract, Simon, a working-class student, talked about the way in which he opted out of the *game* of impressing the mathematics teacher:

> S: Yes and in a way right, when I came to the school, I was scared to ask questions man, so I just thought, no forget it man. (Simon, AH, Year 11, Set 7)

Simon's withdrawal due to fear probably served to disadvantage him when setting decisions were made. The disproportionate allocation of working-class students to low sets shown by the correlations at Amber Hill would certainly have restricted the achievement of working-class students. However, it seems likely that students' social class also may have affected the way in which individuals responded to the experiences of setted lessons. In the next section, I attempt to draw together the various results reported so far to illuminate the different factors that influence student's achievement in setted and mixed-ability groups.

In any debate about the implications of setted and mixed-ability grouping, it is important to consider students' achievement. The approaches of Amber Hill and Phoenix Park schools differed in many important ways, but the GCSE results reported in chapter 6 show that the setted classes did not achieve better results than the students in the mixed-ability classes despite the increased time Amber Hill students spent working. The students who learned mathematics in an open environment in mixed-ability classes achieved significantly more A to G grades despite the comparability of the two cohorts of students on entry to their schools.

DISCUSSION AND CONCLUSION

In concluding this chapter, I consider the implications of the data reported for decisions about mathematics ability grouping in the United States. The negative reports that students gave at Amber Hill—of limited opportunities through the restrictions that setting placed on attainment and unproductive learning environments created by *setted expectations*—are also features of tracked classrooms in the United States. These are not necessary outcomes of tracked systems, but research tells us that they are sufficiently frequent for us to take them very seriously. Additionally, research has repeatedly demonstrated that the most important factor in determining student success is opportunity to learn (Porter & Associates, 1994). Put simply, if students are not exposed to high-level content, they cannot learn it. The United States and England are two of the most highly tracked educational systems in the world, both operating a fine level of sorting that ensures certain students are never exposed to high-level content, particularly in mathematics. Neither of the two countries fare particularly well in international comparisons, and one of the main results of international comparisons is that those countries that *track the least and the latest* have the highest overall achievement (Burstein, 1993). Various forms of evidence align to cast doubt on the effectiveness of ability grouping, certainly enough for us to think carefully about alternatives to this practice, for many of the reasons that have emerged from Amber Hill's data.

Phoenix Park's mathematics teachers achieved something that many people think is not possible. They taught a wide range of students together in the same classes and provided stimulating and appropriate experiences for each (or most) of them. The teachers did this by paying careful attention to the different learning experiences the students needed. I have witnessed attempts in the United States and England to teach mixed-ability classes in exactly the same way as people teach tracked classes—by delivering the same content to everyone and expecting everyone to keep up and learn. Such an approach is unlikely to result in productive learning and will probably be as disadvantageous as a system of severe tracking. Teachers of mixed-ability classes have to provide differentiated work. There are two different ways of doing this: differentiating by task or differentiating by outcome. The former means that different work is available for different students; the latter means that students begin with the same activities, but that such activities are sufficiently open to enable students to approach them in a number of ways. The Phoenix Park teachers employed the latter approach, with some aspects of the former—when they steered different students toward different tasks. In my own experiences as a secondary school mathematics teacher in England, I employed both approaches, and found them both to be successful, enabling appropriate learning experiences for high and low attaining students. I would also add that they both take carefully prepared lessons. In England, a teaching scheme called "SMILE mathematics" has been created by a group of teachers over many years; it provides a vast amount of differentiated materials for students of different attainment levels, in support of mixed ability teaching (see www.smilemathematics.co.uk). Differentiation is something I rarely see in U.S. classrooms, and some teachers with whom I work are resistant to the idea that different students need different materials because this requires additional planning. But differentiation may be an important part of maximizing success. The Phoenix Park approach involved the teachers planning the content students would need to meet and choosing a careful range of open-ended activities that would allow them to meet it in different ways. However, their success with the students involved much more than the selection of appropriate materials. The teachers also needed to provide appropriate scaffolding through help, questioning, and instruction. Each of the Phoenix Park teachers had a good sense of what their students were capable of and used this information in supporting students as they worked on their projects. One of their rewards for this complex undertaking was students who came to them with low grades—who in most schools would have been destined for a low set, low achievement, and a nonmathematical career—attaining the highest possible grades in the examination. This gives some indication of the potential reward for the additional investments that teachers may need to make in planning differentiated opportunities for students in mixed-ability groups.

Ideas about student achievement and grouping are deeply cultural (Stigler & Hiebert, 1999). In Sweden, ability grouping is illegal because it enhances inequality, and there is no more reason needed. In Japan, student ability grouping is similarly avoided. A student at Stanford University (Yiu, 2001) recently interviewed some Japanese mathematics teachers who explained why they do not use ability grouping in the following terms:

> In America being special is good. In Japan what is important is balance. Everyone can do everything, we think that is a good thing. Everyone being the same is good, we are very comfortable this way. So we can't divide by ability. (Teacher A)

> Japanese education emphasizes group education, not individual education. Because we want everyone to improve, promote and achieve goals together, rather than individually. That's why we want students to help each other, to learn from each other . . . , to get along and grow together—mentally, physically and intellectually. (Teacher B)

In England and the United States, inequality is accepted more easily. Japan and Sweden are both highly successful countries in international comparisons, but ability grouping comes so naturally to the majority of British and American mathematics teachers that such facts hardly give cause for pause. I conclude this chapter by reiterating that the Amber Hill approach differentiated students' opportunities in negative ways—ways that are prevalent in the U.S. system. For example:

- Social class influenced setting decisions, resulting in disproportionate numbers of working-class students being allocated to low sets.

- Significant numbers of students experienced difficulties working at the pace of the class, resulting in disaffection and reported underachievement.
- Students became disillusioned and demotivated by the limits placed on their achievement within their sets.
- Some students responded badly to the pressure and competition of setted lessons, particularly girls and students in top sets.

The additional work in which the Phoenix Park teachers engaged—selecting activities on which students could work at different levels and providing concomitant support—impacted many of their students' mathematical understandings in important and positive ways.

The consequences of tracking and setting decisions are great. Indeed, the set or track that students are placed into at a very young age almost certainly dictates the opportunities they receive for the rest of their lives. It is now widely acknowledged in educational and psychological research that students do not have a fixed ability that it is determinable at an early age. However, the placing of students in academic groups often results in the fixing of their potential achievement. Slavin (1990) makes an important point in his review of research in this area. He notes that, because mixed-ability teaching is known to reduce the chances of discrimination, the burden of proof that ability grouping is preferable must lie with those who claim that it raises achievement. Despite the wide range of research studies in this area, this proof has not been forthcoming.

11

Looking to the Future

In this book, I have related the teaching and learning experiences of students in two schools to the knowledge, beliefs, and understandings they developed as evidenced in a range of assessments. There are a number of theoretical perspectives that might be used to explain or interpret the findings from these schools. For example, the two approaches could be taken as examples of constructivist and nonconstructivist teaching. I have chosen to analyze the results from a situated perspective because this provided a framework that enabled me to address the ways in which individuals dealt with different situations. The breadth of this framework was fundamental in understanding why students used mathematics in one setting and not another; why they appeared to have knowledge, but they did not always choose to use it; and how their learning practices came to influence their practices elsewhere. The findings of this study, interpreted within this framework, illustrate the inherent complexity of the learning process—and, crucially, that it is wrong to believe that assessments merely indicate whether a student has *more* or *less* knowledge. Evaluations and analyses of mathematics teaching and learning need to include consideration of the different forms of knowledge that learners develop, the practices in which they engage as learners, and the relationships that such practices afford with the discipline of mathematics (Boaler & Greeno, 2000).

At Amber Hill, many of the students appeared to be disadvantaged in the face of new or applied situations. This seemed to be due to a combination of the students' perceptions about mathematics, their understanding of mathematics, and the goals they formed in different settings. The Amber Hill students believed that mathematical success required memory rather than thought. Many students developed a shallow and procedural knowledge that was of limited use in new and demanding situations, and their desire and need to find and follow cues suppressed their ability to interpret situations holistically or mathematically. Lave (1988) has proposed that notions of transfer cannot explain the way individuals use knowledge in different settings because transfer theories do not take account of the *communities of practice* in which people operate. The results of this study support this idea on a number of different levels. For example, the Amber Hill students regarded the mathematical classroom as a highly specialized community of practice unrelated to all others. This view was formed in response to various aspects of their school setting, such as the formalized nature of the mathematics they encountered, lack of social interaction in their classrooms, and imposition of school rules. These all encouraged the students to locate their mathematical knowledge within the four walls of their mathematics classrooms. The different classroom practices that I termed *cue based* (Schoenfeld, 1985) were used by the students to determine choice of mathematical method. The students became proficient at finding and interpreting different cues within their mathematics textbooks, which helped them proceed through exercises. These cues developed in response to the norms of the classrooms, but in many ways they were the antithesis of mathematical thinking. In a range of different applied assessments, the Amber Hill students became confused because they tried to employ the same cue-based practices they had developed in their mathematics classrooms and found that they were ineffective in nontextbook situations. Thus, their mathematical competence in different situations and the extent to which they were able to transfer was not just about the knowledge they held, but the practices they had developed as mathematics learners.

In the real world, students reported that they did not even attempt to make use of school-learned methods—not because of the form or structure of the mathematical problems they encountered in the real world (Masingila, Davidenko, & Prus-Wisniowska, 1996), but because the environments of the classroom and their everyday lives were too disparate, as one student described:

> G: I use my own methods. . . . 'Cause when we're out of school yeah, we think, when we're out of school it's *social*, you're not like in school, it tends to be *social*, so it would be like too much change to refer back to here. (George, Amber Hill, Year 10)

In this extract, George offers the social differences between the environments of school and the real world as instrumental in his choice of mathematical method. The Amber Hill teachers explained mathematical methods clearly, and the students received opportunities to practice these methods. The students were confident in their use of school methods in the classroom, and it is inconceivable to think that they understood none of these methods. Yet the students suggested that they were unable to use any of their school mathematical methods in real-world situations. Further, the reasons they gave for this were not related to inadequate understanding, but the differences in the environments of school and the real world, and the different practices in which they engaged in the two places (Boaler, 1996a, 1996b).

At Phoenix Park, the boundaries between school and the real world were less distinct. This appeared to stem from a number of features of the school's approach, including (but not only) the activities of the mathematics classroom and the forms of engagement they encouraged. At Phoenix Park, students needed to interpret mathematical situations, choose methods, adapt methods, and solve problems. When they entered other mathematical situations, such as those in the real world and the examination, they readily engaged in similar *practices*. The knowledge they had developed was important, but the practices in which they engaged are also worthy of consideration in their own right. In classrooms such as those of Amber Hill, students learn to repeat procedures and follow rules. They learn to interpret cues and depend on nonmathematical aspects of questions for their choice of procedures. When they move into other mathematical situations, students try to use the same strategies, often with limited success. The central message of situated theory is a simple one—pedagogies matter. They are not just vehicles for more or less knowledge, they come to define the knowledge that is produced. This is partly because students not only learn mathematics knowledge in classrooms they learn particular mathematical practices that are differentially useful in different situations.

I have reemphasized the relationship between knowledge and practice that is highlighted by a situated lens in order that those considering the validity of different curriculum approaches may consider the practices as well as the knowledge communicated through different approaches. Put simply, if we want students to consider mathematical situations and flexibly make use of mathematics knowledge in the real world or in examinations of higher mathematics, we need to engage students in similar practices in the classroom. It is through such practices that students will develop identities as mathematics problem solvers.

I started this chapter by focusing on the practices in mathematics classrooms and the identities students develop through these, partly in acknowledgment of the complexity of teaching and learning. Many of the beliefs and understandings students developed at the two schools were a function of their classroom communities, but these were not explicitly taught to students. They emerged over time through the classroom interactions in which teachers and students engaged. Many important characteristics combined to produce these environments at both schools. I have described some of these in this book and in other publications (Boaler, 1997a, 1997b, 1997c, 1998, 1999, 2000b, 2002a), and I will not repeat all of those now. Rather, I will spend a little time describing the features that seemed most important to the students' mathematical effectiveness at Phoenix Park.

IMPORTANT FEATURES OF PHOENIX PARK'S APPROACH

Students at Phoenix Park worked on mathematics problems that teachers had chosen to engage students. The problems all required considerable mathematical thought, and they enabled students to work in different directions and at different levels. Some of the problems involved real-world contexts, but many did not. The teachers had collected the different problems over a number of years, collecting ideas from a range of publications, teacher meetings, and conferences. They worked as a department to choose the problems, attending to the mathematical content that students needed to meet, as well as the practices in which they hoped students would engage, such as communicating mathematical ideas, justifying solutions, and adapting different methods.

In the classroom, the teachers worked to make the problems interesting and meaningful for students. They adapted different problems for different students, and they helped students navigate their way through the problems. When the students encountered difficulties, the teachers did not tell them what to do; they asked them questions that encouraged them to think and make connections between the problems on which they were working and the mathematical methods they had learned.

Over the years that students worked on the problems, they engaged in a range of pedagogical practices. For some of the time, they would work alone; at other times, they worked in groups. The students employed technological resources such as computers and calculators at times, but at other times they did not because they were encouraged only to use them when it was appropriate. The students spent some time using methods they had been taught; at other times, they adapted methods in applying them to new situations. Sometimes the teacher would introduce ideas to the whole class; at other times, they would introduce them to pairs or groups. Sometimes the students engaged in whole-class discussions (infrequently); at other times, they talked with a partner or group. The *range* of practices in which students engaged was important because the students did not come to develop a restricted view of mathematical activity as the Amber Hill students did.

The Phoenix Park teachers employed a number of different roles in the classroom that required a good understanding of mathematics and students, but the teachers were not exceptional in either regard. Rosie was in her first year of teaching, but she learned a lot from her experiences in the classroom, quickly developing different teaching ideas from her colleagues and students. The different teachers in the department provided a great deal of support for each other, and the administration of the school was also relatively supportive. What was unusual about the teachers was that they believed that *all* students could achieve at high levels, and they were all committed to equity. The teachers pushed all of their students, and they believed that students would be encouraged to think and learn if teachers refrained from structuring their mathematical experiences too much. This commitment meant that if students were underachieving, the teachers paid more careful attention to what they thought students needed to learn to achieve. Some teachers believe that students who experience more difficulty should be given more structure (Confrey, 1990; Orton & Frobisher, 1996). This idea is easy to understand, particularly for those of us who have been in teaching situations when a student has expressed frustration at trying to understand a concept and the provision of a structured procedure would have encouraged immediate success. But my observations of teaching and learning in high- and low-attainment groups, and interviews with students in these groups (Boaler, Wiliam, & Brown, 2001), have helped me understand the importance of questioning the relationship between mathematical level and structure.

Additionally, the Phoenix Park teachers demonstrated that students of all levels and all classes could develop a conceptual understanding of the mathematics with which they were engaged. They did not succumb to the temptation of spoon-feeding those students who sought such help, and the rewards of their hard work were demonstrated by the students' achievements. This is not to suggest that teachers should never make decisions to provide students with additional structure, only that such decisions should not correlate with mathematical level or social class. As long as we hold conceptual understanding as a goal for students, it is imperative that such a goal is held for *all* students. Awareness that students of low social class (Lubienski, 2000) or achievement encounter difficulties interpreting open work must be accompanied by a drive to understand the students' experiences better and provide action to make the teaching of open-ended approaches more equitable (Ball, 1995; Boaler, 2002a). Mathematics teachers often believe that some students are not capable of learning at high levels (Gutiérrez, 1996). For the Amber Hill teachers, this belief intersected with their ideas about social class. The Phoenix Park teachers believed that all students were capable of high-level mathematical attainment if they received appropriate encouragement. The teachers were also unusual in believing that students would learn a great deal if they were asked probing questions, rather than being shown elegant solutions that they were expected to reproduce. In many instances, when other teachers may have shown students what to do, the Phoenix Park teachers asked questions and left the students to think about them before they would intervene again.

There is not the space in this book to describe all the mathematical activities the Phoenix Park teachers used or all the practices in which they engaged in the classroom. Therefore, this book will not provide the information that other teachers may need to enact a similar approach (if any book could). What I hope this book conveys is a sense of what is possible. Along with this, I hope this research has furthered understanding of the relationship between different classroom interactions and the understandings, beliefs, and dispositions students develop. The results of this study do not show that all reform approaches are *best*, only that they can produce powerful understandings among students if taught well and are worthy of further understanding and investment. Many antireform lobbyists in the United States go to great lengths to stop open-ended approaches to learning because they fear they will leave students ill prepared. Such fears are understandable because open-ended approaches do require a lot of teachers. But if similar energies were spent helping teachers become better prepared, then it seems likely that the nation's children would be considerably better served. Procedural teaching approaches have served few students well in the past—offering limited opportunities for understanding, identification, and affiliation with mathematics. Open approaches hold the potential for powerful understandings and engagement, as illustrated by the achievements of the Phoenix Park students and teachers, but their success in schools across the United States will require the support, rather than denigration (Jacob, 2001) of teachers who are working to use them to good effect.

LOOKING TO THE FUTURE

Stephen Ball (1993) described the conservative vision for education as one in which desks are "in rows, the children silent, the teacher at the front, chalk in hand, dispensing knowledge" (p. 209). This vision, which is consistent with the wishes of some antireformers in the United States, was perfectly represented by the mathematics classrooms at Amber Hill. In these classrooms, there was an emphasis on order and control, the learning of specified, mathematical methods, "chalk and talk" transmission teaching, with children divided into eight narrow bands of "homogeneous" ability. This research study has demonstrated that each of these traditional features of Amber Hill's mathematics teaching disadvantaged some students in some ways. This was not because the teachers were incompetent or lacked commitment. It arose from the pedagogical, philosophical, and epistemological models embraced by the teachers. For the teachers at Amber Hill believed in giving students structured pieces of mathematical knowledge to learn—in line with what Ball (1993) called the "curricular fundamentalism" of the conservatives (p. 205). The teachers did not perceive a need to give students the opportunity to think about, use, or discuss mathematics. Sigurdson and Olson (1992) note that many mathematics teachers consider learning and understanding to be synonymous; because of this, much school learning is done at a rote level. The Amber Hill teachers conformed with this model—they did not see any real difference between a clear transmission of knowledge and student understanding. Most of the problems experienced by the Amber Hill students derived from this knowledge transmission approach—a central feature that shaped mathematics teaching at the school. Other traditional features of the students' environment, such as setting and high-pressure learning, served to exacerbate their problems, but it was the transmission of closed pieces of knowledge that formed the basis of much of the students' disaffection, misunderstandings, and underachievement.

The term *progressive* is a label often used in a pejorative way to describe supposedly ineffective teaching approaches. The Phoenix Park approach was based on principles of independence and self-motivation, and such a label does not begin to reflect the complexity of the different characteristics that constituted the school's approach. However, I have chosen to adopt this term to describe the combination of the school's different features partly to juxtapose the Phoenix Park approach with the back-to-basics movement and partly because Phoenix Park school embraced many of the principles that traditionalists most fear when they talk about progressive education. At Phoenix Park, the students were schooled in a totally different way than the students at Amber Hill. Although the most obvious result of the school's progressivism and lack of imposed order was classrooms that many would describe as chaotic, the results from this study show that the students learned more effectively than the Amber Hill students. The Phoenix Park students reported that they developed self-motivation and self-discipline as a result of the school's approach, that the openness of their work encouraged them to think for themselves, and the need to use mathematics in different activities caused them to be adaptable and flexible in their approach to mathematics.

I do not wish to imply that Phoenix Park represented an ideal learning environment; it clearly did not, but a consideration of the ways in which lessons could have been improved did not suggest a move toward the Amber Hill model of teaching. For example, limited classroom observations might suggest that more of the Phoenix Park students could be encouraged to work, but my observations of the students at both schools showed quite clearly that merely making them work did not improve their learning. Phoenix Park students worked when they chose to, but they still achieved more than the disciplined students at Amber Hill. This suggests that the most important aim for teachers should be to engage students and provide worthwhile activities that they find stimulating. This is supported by the work of Bell (1993), who found that intensity and degree of engagement were more important than time on task. High levels of intensity are impossible to maintain all the time, but the Phoenix Park students at least experienced real engagement for some of their school lives. When Mickey and Ahmed (see chap. 5) reported in chapter 5, discovered the way in which they could use trigonometry to find an area, they were genuinely interested and excited. The contrast between this and the Amber Hill students' learning of trigonometry could not be more extreme. The findings of this research indicate that lessons in both schools would be improved if students experienced this sort of excitement and engagement more often. But the key to this improvement has to be the design of appropriate activities and the creation of stimulating work environments, not a simple increase in discipline and order.

Mathematics education has recently taken a leading role within public discussions in response to claims of falling standards, poor performance in international studies, and badly prepared university students (Becker & Jacob, 2000). Such reports have reopened debates about the relative advantages of traditional, "back to basics" approaches to teaching versus the "reform" methods, which are commonly cited as culprits in these accounts. Yet these debates rarely draw on longitudinal evidence of student learning. Amber Hill's mathematics approach was not unusual, as supported by a large body of research (Peterson & Fennema, 1985; Romberg & Carpenter, 1986). Peterson (1988) reports that the majority of mathematics teaching is focused on the teaching and learning of basic facts and algorithmic procedures. Cheek and Castle (1981) question whether the term "back to basics" can be applied to mathematics education when evidence shows that a basic approach was never abandoned by the majority of mathematics teachers. They point to research that has shown that "mathematics instruction has changed little over the past 25 years, despite the innovations advocated" (p. 264), and that a single textbook continues to be the main source of content in mathematics lessons, with the majority of instruction occurring from the front, followed by the rehearsal of methods in numerous exercises. Inspections from Her Majesty's Inspectorate (HMI) in England have shown that most teachers are essentially cautious and conservative (Bolton, 1992), and various forms of evidence indicate that this description can be more accurately applied to teachers of mathematics than any other subject group. All of this leads to the conclusion that, if mathematics performance is lower than that of other subjects, this is more likely to be due to the traditionalism rather than the progressivism of mathematics teachers.

The various findings of this study offer a bleak view of the Amber Hill students' learning, but the research evidence reviewed earlier suggests that the Amber Hill approach is fairly typical for high school mathematics. My observations of mathematics classrooms over the last 15 years would support this. Jaworski (1994) also notes that in 12 years of teaching mathematics in different parts of England, the "exposition and practice" approach (p. 8) was the most common. If the Amber Hill teachers were particularly unusual, it would seem unlikely that all eight of the teachers in the department would share the same unusual characteristics, yet the eight different mathematics teachers who varied in popularity and experience prompted the same set of responses from students. The only distinctive feature that I noted at Amber Hill was the teachers' tendency to make mathematics even more closed and rule bound because of the students' working-class background. This tendency to move mathematics into a closed domain served to demonstrate even more clearly the implications of such an approach for the mathematics learning of students.

The findings of this study should also prompt consideration of the value of the narrow, closed assessments used to measure mathematics capability in England and the United States. At Phoenix Park, the school was successful in giving students a broad perspective on mathematics, and the students had become open, flexible thinkers. All this changed when they reached Christmas of Year 10 and they started examination preparation. At this time, they narrowed their view of mathematics—they thought the new mathematical procedures they were learning were confusing and irrelevant, and they constructed barriers or boundaries (Lave, 1996; Siskin, 1994) between the mathematical knowledge of the classroom and the mathematical demands of their jobs and lives. Lerman (1990) states that new forms of learning require new forms of assessment, and it was obvious that the Phoenix Park students were disadvantaged by an examination system that was incompatible with their school's approach, although they attained higher grades than the Amber Hill students. More generally, the demands on teachers to prepare students for examinations that assess methods and procedures, in narrow and closed questions, diminishes the potential for teachers to move away from a narrow and closed teaching model and reduces the likelihood of their spending time letting students explore and use mathematics in open or authentic situations.

Prior to the start of my research study, Phoenix Park was involved in a pilot of a new examination that combined open and closed questions to assess mathematical process as well as content. In 1994, the School Curriculum and Assessment Authority withdrew this new form of GCSE examination. The next cohort of Phoenix Park students was required to take the more traditional closed examination. The proportion of students attaining grades A to C and A to G shifted from 32% and 97%, respectively, in 1993 to 12% and 84%, respectively, in 1994. In the summer following the end of my 3-year research project, Phoenix Park was inspected by a new group of government inspectors called the Office of Standards in Education (OFSTED). This group of inspectors was set up by the Conservative party and is different from Her Majesty's Inspectors. The OFSTED inspectors were led by a man who constantly encouraged transmission models of teaching. In anticipation of this inspection and the need to increase GCSE grades, the head teacher at Phoenix Park forced the mathematics department to end their project-based approach and teach from textbooks. In response to the new middle-class parents putting pressure on the school, Phoenix Park also started to place students into ability groups for mathematics. The teachers in the mathematics department responded badly to these changes with feelings of demoralization and disempowerment. Jim Cresswell was convinced that the students were being disadvantaged in many ways, and that the changes would not increase examination performance, particularly for students in low set groups who, he reported, had become disaffected. Jim believed that he was ineffective as a textbook teacher and has now left the teaching profession. Significantly, he believed that there was no place for an open, authentic approach to mathematics education within the "back to basics" climate of the time.

I have made some fairly strong and controversial claims in this book, and questions are bound to be raised about their generalizability. Some may argue that the disaffection and underachievement that the students experienced at Amber Hill were related to intrinsic features of Amber Hill school, and can therefore be ignored. Questions about the generalizability of the study should, of course, be raised, but I hope that the detail of the study and the students' own accounts of their learning will provide readers with sufficient information to base their decisions about factors of importance at the two schools. This is part of the value of ethnographic accounts: They do not provide multiple instances of the same phenomenon, but they do provide the detail for readers to decide for themselves about the relevance of the reported experiences to their own settings. In conducting this study, I became convinced that the disadvantages faced by Amber Hill students were not specific to that school. I also became convinced that it would be wrong to ignore the messages given by the Amber Hill and Phoenix Park students. The scarcity of the type of mathematics environment encouraged at Phoenix Park make the students' reported experiences from this school particularly important. The messages that emerged from the two schools go against the current tide of antireform public opinion and associated drives to increase the formalization of mathematics teaching, but this must surely make these messages all the more important to consider.

My main concern in this study has been with the nature and form of the practices in which students engage in classrooms and their relationship with knowledge. To investigate these, I employed both qualitative and quantitative methods, and my claims for rigor rest on the triangulation of many different forms and sources of data. The classroom observations I conducted at the two schools over 3 years and my subsequent coding and analysis of these data, along with other forms of data such as interviews and assessments, took a vast amount of time, but they were essential in understanding the relationship between classroom practices and student achievement. Most of those who oppose reform approaches (Becker & Jacob, 2000) in the United States have not seen the approaches enacted in classrooms, let alone studied their enactment systematically over some years. Since first publishing this study, I have been contacted by some of the mathematicians who firmly oppose reforms in England and the United States; they have asked for the real names of the schools so that they may visit them and confirm or disconfirm my findings. I cannot provide the names of the schools in this study because teachers and students in research studies are promised anonymity. This is a standard ethical practice to protect research subjects from any harm. But even if I had been able to, a visit from a skeptic would not constitute research and would not compare to a 3-year analysis of the learning of a matched sample of 300 students. Since communicating that message to the mathematicians in question, they have responded in various ways. One wrote to me telling me not to communicate the results of my research; another called me to his office at Stanford to tell me the same message. A third has recently written to a Web forum saying that I invented the whole study ("The schools exist in only her mind"). Such responses are characteristic of a number of features of the "math wars," including the complete dismissal of evidence that goes against opinion and lack of consideration of the ways that *particular* teaching decisions impact learning. Such responses are certainly antiintellectual and unscholarly in their suppression of research evidence, but they are also dangerous in their lack of willingness to consider any forms of teaching and learning that depart from the familiar—even when such methods have not served the majority of students well.

The students who left Amber Hill and Phoenix Park at the end of my research study had developed different capabilities and understandings as a result of their school experiences. At Amber Hill, many of the students were submissive, unlikely to think mathematically in situations they would encounter, and generally disillusioned by their mathematical experiences. At Phoenix Park, many of the students were confident, liked to use their initiative, and were flexible in their use of mathematics. These responses can be related back to the mathematical and whole school approaches they experienced. Phoenix Park's mathematics department has now moved a long way toward the Amber Hill model of teaching, and there is evidence that many other schools are returning to policies of ability grouping and textbook teaching in response to government initiatives in England. Perhaps the most worrying result of this trend is that there no longer seems to be a place in schools for teachers who want to innovate or try new approaches or strive toward something more than test training. Jim was forced to leave teaching because he did not know of any school that taught mathematics using an open approach despite the enormous wealth of research evidence, spanning over 60 years, that has shown the advantages of these approaches (Baird & Northfield, 1992; Benezet, 1935a, 1935b, 1936; Charles & Lester, 1984; Cobb, Wood, Yackel, & Perlwitz, 1992). Phoenix Park's open, project-based approach has been eliminated, and there is a real possibility that the students who left the school in 1995 as active mathematical thinkers will soon be replaced by mathematics students who are submissive and rule-bound and who see no use for the methods, facts, rules, and procedures they learn in their school mathematics lessons:

Sue: If we do use maths outside of school it's got the same atmosphere as how it used to be, but not now.

JB: What do you mean by it's got the same atmosphere?

Sue: Well, when we used to do projects, it was like that, looking at things and working them out, solving them—so it was similar to that, but it's not similar to this stuff now, it's, you don't know what this stuff is for really, except the exam. (Sue, Phoenix Park, Year 10)

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