

Cálculo 2 - 2025.1

Aulas 46 e 47: EDOs exatas

Eduardo Ochs - RCN/PURO/UFF

<http://anggtwu.net/2025.1-C2.html>

Links

[StewPtCap14p25](#) (p.811) 14.3 Derivadas Parciais
[StewPtCap14p45](#) (p.831) 14.5 A Regra da Cadeia
[StewPtCap14p47](#) (p.833) A Regra da Cadeia (versão geral)
[BoyceDip2p54](#) (p.72) 2.6 Equações Exatas e Fatores Integrantes
[BoyceDip2p58](#) (p.76) Problemas
[BoyceDipEng2p50](#) (p.70) 2.6 Exact Differential Equations and Integrating Factors
[BoyceDipEng2p55](#) (p.75) Problems
[ZillCullenCap2p25](#) (p.60) 2.4 Equações exatas
[ZillCullenCap2p32](#) (p.67) Exercícios
[ZillCullenEngCap2p35](#) (p.62) 2.4 Exact equations
[ZillCullenEngCap2p41](#) (p.68) Exercises 2.4
[DiffyQsP63](#) 1.8 Exact Equations
[DiffyQsP70](#) 1.8.3 Exercises
[2yT14](#) (2019.2) P2
<http://angg.twu.net/LATEX/2019-2-C2-P2.pdf>
[2jQ97](#) (2024.2) Quadros das aulas sobre EDOs exatas
[2iQ93](#) (2024.1) Quadros das aulas sobre EDOs exatas
[2iQ93](#) (2024.1) Quadros das aulas sobre EDOs exatas
[2hQ73](#) (2023.2) Quadros das aulas sobre EDOs exatas
[2yQ106](#) (2019.2) Quadros das aulas sobre EDOs exatas

Método e exemplo

$$[E_5] = \begin{pmatrix} dz = z_x dx + z_y dy = 0 \\ \frac{d}{dx} z = z_x + z_y \frac{dy}{dx} = 0 \\ z = C \end{pmatrix}$$

$$[E_3] = \begin{pmatrix} z_x dx + z_y dy = 0 \\ z_x + z_y \frac{dy}{dx} = 0 \\ z = C \end{pmatrix}$$

$$[E_2] = \begin{pmatrix} z_x + z_y \frac{dy}{dx} = 0 \\ z = C \end{pmatrix}$$

$$[S_1] = \begin{bmatrix} z := (x^2 y^3) \\ z_x := (2xy^3) \\ z_y := (3x^2 y^2) \end{bmatrix}$$

$$[S_2] = \begin{bmatrix} z := (x^2 + y^2) \\ z_x := (2x) \\ z_y := (2y) \end{bmatrix}$$

$$[E_5][S_1] = \begin{pmatrix} d(x^2 y^3) = (2xy^3)dx + (3x^2 y^2)dy = 0 \\ \frac{d}{dx}(x^2 y^3) = (2xy^3) + (3x^2 y^2) \frac{dy}{dx} = 0 \\ (x^2 y^3) = C \end{pmatrix}$$

$$[E_3][S_1] = \begin{pmatrix} (2xy^3)dx + (3x^2 y^2)dy = 0 \\ (2xy^3) + (3x^2 y^2) \frac{dy}{dx} = 0 \\ (x^2 y^3) = C \end{pmatrix}$$

$$[E_2][S_1] = \begin{pmatrix} (2xy^3) + (3x^2 y^2) \frac{dy}{dx} = 0 \\ (x^2 y^3) = C \end{pmatrix}$$

$$[E_5][S_2] = \begin{pmatrix} d(x^2 + y^2) = (2x)dx + (2y)dy = 0 \\ \frac{d}{dx}(x^2 + y^2) = (2x) + (2y) \frac{dy}{dx} = 0 \\ (x^2 + y^2) = C \end{pmatrix}$$

$$[E_3][S_2] = \begin{pmatrix} (2x)dx + (2y)dy = 0 \\ (2x) + (2y) \frac{dy}{dx} = 0 \\ (x^2 + y^2) = C \end{pmatrix}$$

$$[E_2][S_2] = \begin{pmatrix} (2x) + (2y) \frac{dy}{dx} = 0 \\ (x^2 + y^2) = C \end{pmatrix}$$

Uma questão da P2 de 2019.2

4) Sejam (***) e (****) estas EDOs:

$$2xy^3 dx + 3x^2y^2 dy = 0 \quad (***)$$

$$2x^2y^3 dx + 3x^3y^2 dy = 0 \quad (****)$$

- a) **(0.5 pts)** Mostre que (***) é exata.
- b) **(0.5 pts)** Encontre a solução geral de (***) .
- c) **(1.0 pts)** Teste a sua solução geral da (***) .
- d) **(0.5 pts)** Mostre que a solução geral da EDO (***) também é solução da (****) .
- e) **(0.5 pts)** Mostre que (****) não é exata.
- f) **(0.5 pts)** Mostre que o fator integrante obtido por

$$\begin{aligned} p(x) &= (M_y - N_x)/N, \\ \mu(x) &= e^{\int p(x) dx} \end{aligned}$$

transforma (****) em (***) .

Versão original:

<http://angg.twu.net/LATEX/2019-2-C2-P2.pdf>

```
(%i1) load("2025-1-exact.mac")$
```

```
(%i2) ee : Exact_from0()$
```

```
(%i3) ee@@E5();
```

```
(%o3)
```

$$\begin{pmatrix} dz & = & z_x dx + z_y dy & = & 0 \\ \frac{d}{dx} z & = & z_x + z_y \left(\frac{d}{dx} y\right) & = & 0 \\ z & = & C \end{pmatrix}$$

```
(%i4) ee : Exact_from1(x^2+y^2)$
```

```
(%i5) ee@@E5();
```

```
(%o5)
```

$$\begin{pmatrix} d(y^2 + x^2) & = & 2x dx + 2y dy & = & 0 \\ \frac{d}{dx}(y^2 + x^2) & = & 2x + 2y \left(\frac{d}{dx} y\right) & = & 0 \\ y^2 + x^2 & = & C \end{pmatrix}$$

```
(%i6) ee : Exact_from2(2*x, 2*y)$
```

```
(%i7) ee@@E5();
```

```
(%o7)
```

$$\begin{pmatrix} dz & = & 2x dx + 2y dy & = & 0 \\ \frac{d}{dx} z & = & 2x + 2y \left(\frac{d}{dx} y\right) & = & 0 \\ z & = & C \end{pmatrix}$$

```
(%i8) ee@@ode();
```

```
(%o8)
```

$$2y \left(\frac{d}{dx} y\right) + 2x = 0$$

```
(%i9) ee@@odesolve();
```

```
(%o9)
```

$$-\left(\frac{y^2}{2}\right) = \frac{x^2}{2} + \%c$$

```
(%i10) sols : ee@@odesols();
```

```
(%o10)
```

$$\left[y = -\sqrt{-x^2 - 2\%c}, y = \sqrt{-x^2 - 2\%c} \right]$$

```
(%i11) sol : sols[1];
```

```
(%o11)
```

$$y = -\sqrt{-x^2 - 2\%c}$$

```
(%i12) test1 : ee@@odesubst(sol);
```

```
(%o12)
```

$$2x - 2\sqrt{-x^2 - 2\%c} \left(\frac{d}{dx} \left(-\sqrt{-x^2 - 2\%c}\right)\right) = 0$$

```
(%i13) test2 : ev(test1, diff);
```

```
(%o13)
```

$$0 = 0$$

```
(%i1) load("2025-1-exact.mac")$
```

```
(%i2) ee : Exact_from1(x^2*y^3);
```

```
(%o2)
```

Exact ($x^2 y^3, 2 x y^3, 3 x^2 y^2$)

```
(%i3) ee@@caixinhas_3();
```

```
(%o3)
```

$$\left[\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right]$$

```
(%i4) ee@@caixinhas_2();
```

```
(%o4)
```

$$\left[\begin{pmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right]$$

```
(%i5) ee : Exact_from1(x^2+y^2);
```

```
(%o5)
```

Exact ($y^2 + x^2, 2 x, 2 y$)

```
(%i6) ee@@caixinhas_3();
```

```
(%o6)
```

$$\left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right]$$

```
(%i7) ee@@caixinhas_2();
```

```
(%o7)
```

$$\left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right]$$