Cálculo 2 - 2025.1

Aulas 46 e 47: EDOs exatas

Eduardo Ochs - RCN/PURO/UFF
http://anggtwu.net/2025.1-C2.html

Links

Método e exemplo

$$[\text{E}_5] = \begin{pmatrix} dz = z_x dx + z_y dy = 0 \\ \frac{d}{dx}z = z_x + z_y \frac{dy}{dx} = 0 \\ z = C \end{pmatrix}$$

$$[\text{E}_5][\text{S}_1] = \begin{pmatrix} d(x^2 y^3) = (2xy^3)dx + (3x^2y^2)dy = 0 \\ \frac{d}{dx}(x^2y^3) = (2xy^3) + (3x^2y^2)\frac{dy}{dx} = 0 \\ (x^2 y^3) = C \end{pmatrix}$$

$$[\text{E}_3] = \begin{pmatrix} z_x dx + z_y dy = 0 \\ z_x + z_y \frac{dy}{dx} = 0 \\ z = C \end{pmatrix}$$

$$[\text{E}_3][\text{S}_1] = \begin{pmatrix} (2xy^3)dx + (3x^2y^2)dy = 0 \\ (2xy^3) + (3x^2y^2)\frac{dy}{dx} = 0 \\ (x^2 y^3) = C \end{pmatrix}$$

$$[\text{E}_2] = \begin{pmatrix} z_x + z_y \frac{dy}{dx} = 0 \\ z = C \end{pmatrix}$$

$$[\text{E}_2][\text{S}_1] = \begin{pmatrix} (2xy^3) + (3x^2y^2)\frac{dy}{dx} = 0 \\ (x^2 y^3) = C \end{pmatrix}$$

$$[\text{S}_1] = \begin{bmatrix} z := (x^2 y^3) \\ z_x := (2xy^3) \\ z_y := (3x^2y^2) \end{bmatrix}$$

$$[\text{S}_2] = \begin{bmatrix} z := (x^2 + y^2) \\ z_x := (2x) \\ z_y := (2y) \end{bmatrix}$$

$$[\text{E}_5][\text{S}_2] = \begin{pmatrix} d(x^2 + y^2) = (2x)dx + (2y)dy = 0 \\ \frac{d}{dx}(x^2 + y^2) = (2x) + (2y)\frac{dy}{dx} = 0 \\ (x^2 + y^2) = C \end{pmatrix}$$

$$[\text{E}_3][\text{S}_2] = \begin{pmatrix} (2x)dx + (2y)dy = 0 \\ (2x) + (2y)\frac{dy}{dx} = 0 \\ (x^2 + y^2) = C \end{pmatrix}$$

$$[\text{E}_2][\text{S}_2] = \begin{pmatrix} (2x) + (2y)\frac{dy}{dx} = 0 \\ (x^2 + y^2) = C \end{pmatrix}$$

Uma questão da P2 de 2019.2

4) Sejam (∗∗∗) e (∗∗∗∗) estas EDOs:

$$2xy^3 \, dx + 3x^2y^2 \, dy = 0 \quad (\ast\ast\ast)$$
$$2x^2y^3 \, dx + 3x^3y^2 \, dy = 0 \quad (\ast\ast\ast\ast)$$

a) **(0.5 pts)** Mostre que (∗∗∗) é exata.
b) **(0.5 pts)** Encontre a solução geral de (∗∗∗).
c) **(1.0 pts)** Teste a sua solução geral da (∗∗∗).
d) **(0.5 pts)** Mostre que a solução geral da EDO (∗∗∗)
também é solução da (∗∗∗∗).
e) **(0.5 pts)** Mostre que (∗∗∗∗) não é exata.
f) **(0.5 pts)** Mostre que o fator integrante obtido por

$$\begin{aligned} p(x) &= (M_y - N_x)/N, \\ \mu(x) &= e^{\int p(x) \, dx} \end{aligned}$$

transforma (∗∗∗∗) em (∗∗∗).

Versão original:
http://angg.twu.net/LATEX/2019-2-C2-P2.pdf

```
(%i1) load("2025-1-exact.mac")$
(%i2) ee : Exact_from0()$
(%i3) ee@@E5();
(%o3)
```

$$\begin{pmatrix} d\,z & = & z_x\,\mathrm{dx} + z_y\,\mathrm{dy} & = & 0 \\ \frac{d}{dx}z & = & z_x + z_y\left(\frac{d}{dx}y\right) & = & 0 \\ z & = & C & & \end{pmatrix}$$

```
(%i4) ee : Exact_from1(x^2+y^2)$
(%i5) ee@@E5();
(%o5)
```

$$\begin{pmatrix} d\,(y^2+x^2) & = & 2\,x\,\mathrm{dx} + 2\,y\,\mathrm{dy} & = & 0 \\ \frac{d}{dx}\,(y^2+x^2) & = & 2\,x + 2\,y\left(\frac{d}{dx}y\right) & = & 0 \\ y^2+x^2 & = & C & & \end{pmatrix}$$

```
(%i6) ee : Exact_from2(2*x, 2*y)$
(%i7) ee@@E5();
(%o7)
```

$$\begin{pmatrix} d\,z & = & 2\,x\,\mathrm{dx} + 2\,y\,\mathrm{dy} & = & 0 \\ \frac{d}{dx}z & = & 2\,x + 2\,y\left(\frac{d}{dx}y\right) & = & 0 \\ z & = & C & & \end{pmatrix}$$

```
(%i8) ee@@ode();
(%o8)
```

$$2\,y\left(\frac{d}{dx}y\right) + 2\,x = 0$$

```
(%i9) ee@@odesolve();
(%o9)
```

$$-\left(\frac{y^2}{2}\right) = \frac{x^2}{2} + \%c$$

```
(%i10) sols : ee@@odesols();
(%o10)
```

$$\left[y = -\sqrt{-x^2 - 2\,\%c}, y = \sqrt{-x^2 - 2\,\%c} \right]$$

```
(%i11) sol : sols[1];
(%o11)
```

$$y = -\sqrt{-x^2 - 2\,\%c}$$

```
(%i12) test1 : ee@@odesubst(sol);
(%o12)
```

$$2\,x - 2\,\sqrt{-x^2 - 2\,\%c}\left(\frac{d}{dx}\left(-\sqrt{-x^2 - 2\,\%c}\right)\right) = 0$$

```
(%i13) test2 : ev(test1, diff);
(%o13)
```

$$0 = 0$$

```
(%i1) load("2025-1-exact.mac")$
(%i2) ee : Exact_from1(x^2*y^3);
(%o2)
```
$$\text{Exact}\left(x^2\,y^3,\,2\,x\,y^3,\,3\,x^2\,y^2\right)$$

```
(%i3) ee@@caixinhas_3();
(%o3)
```
$$\left[\begin{pmatrix}0&0&1&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},\begin{pmatrix}0&2&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},\begin{pmatrix}0&0&0&0\\0&3&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right]$$

```
(%i4) ee@@caixinhas_2();
(%o4)
```
$$\left[\begin{pmatrix}0&2&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},\begin{pmatrix}0&0&0&0\\0&3&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right]$$

```
(%i5) ee : Exact_from1(x^2+y^2);
(%o5)
```
$$\text{Exact}\left(y^2+x^2,\,2\,x,\,2\,y\right)$$

```
(%i6) ee@@caixinhas_3();
(%o6)
```
$$\left[\begin{pmatrix}0&0&0&0\\1&0&0&0\\0&0&0&0\\0&0&1&0\end{pmatrix},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&2&0&0\end{pmatrix},\begin{pmatrix}0&0&0&0\\0&0&0&0\\2&0&0&0\\0&0&0&0\end{pmatrix}\right]$$

```
(%i7) ee@@caixinhas_2();
(%o7)
```
$$\left[\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&2&0&0\end{pmatrix},\begin{pmatrix}0&0&0&0\\0&0&0&0\\2&0&0&0\\0&0&0&0\end{pmatrix}\right]$$