Cálculo 2 - 2025.1

Aulas 46 e 47: EDOs exatas

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http://anggtwu.net/2025.1-C2.html

Links

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StewPtCap14p45 (p.831) 14.5 A Regra da Cadeia
StewPtCap14p47 (p.833) A Regra da Cadeia (versão geral)

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BoyceDip2p55 (p.73) Começamos integrando
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Provas:
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2iT227 (2024.1) P2
2yT14 (2019.2) P2

Quadros:
2jQ97 (2024.2)
2iQ93 (2024.1)
2iQ93 (2024.1)
2hQ73 (2023.2)
2yQ106 (2019.2)

Método e exemplo

$$[\mathrm{E}_5] = \left(\begin{array}{rcrcl} dz & = & z_x dx + z_y dy & = & 0 \\ \frac{d}{dx} z & = & z_x + z_y \frac{dy}{dx} & = & 0 \\ z & = & C & & \end{array}\right)$$

$$[\mathrm{E}_3] = \left(\begin{array}{rcrcl} z_x dx + z_y dy & = & 0 & & \\ z_x + z_y \frac{dy}{dx} & = & 0 & & \\ z & = & C & & \end{array}\right)$$

$$[\mathrm{E}_2] = \left(\begin{array}{rcrcl} z_x + z_y \frac{dy}{dx} & = & 0 & & \\ z & = & C & & \end{array}\right)$$

$$[\mathrm{S}_1] = \left[\begin{array}{rcl} z & := & (x^2y^3) \\ z_x & := & (2xy^3) \\ z_y & := & (3x^2y^2) \end{array}\right]$$

$$[\mathrm{S}_2] = \left[\begin{array}{rcl} z & := & (x^2+y^2) \\ z_x & := & (2x) \\ z_y & := & (2y) \end{array}\right]$$

$$[\mathrm{E}_5][\mathrm{S}_1] = \left(\begin{array}{rcrcl} d(x^2y^3) & = & (2xy^3)dx + (3x^2y^2)dy & = & 0 \\ \frac{d}{dx}(x^2y^3) & = & (2xy^3) + (3x^2y^2)\frac{dy}{dx} & = & 0 \\ (x^2y^3) & = & C & & \end{array}\right)$$

$$[\mathrm{E}_3][\mathrm{S}_1] = \left(\begin{array}{rcrcl} (2xy^3)dx + (3x^2y^2)dy & = & 0 & & \\ (2xy^3) + (3x^2y^2)\frac{dy}{dx} & = & 0 & & \\ (x^2y^3) & = & C & & \end{array}\right)$$

$$[\mathrm{E}_2][\mathrm{S}_1] = \left(\begin{array}{rcrcl} (2xy^3) + (3x^2y^2)\frac{dy}{dx} & = & 0 & & \\ (x^2y^3) & = & C & & \end{array}\right)$$

$$[\mathrm{E}_5][\mathrm{S}_2] = \left(\begin{array}{rcrcl} d(x^2+y^2) & = & (2x)dx + (2y)dy & = & 0 \\ \frac{d}{dx}(x^2+y^2) & = & (2x) + (2y)\frac{dy}{dx} & = & 0 \\ (x^2+y^2) & = & C & & \end{array}\right)$$

$$[\mathrm{E}_3][\mathrm{S}_2] = \left(\begin{array}{rcrcl} (2x)dx + (2y)dy & = & 0 & & \\ (2x) + (2y)\frac{dy}{dx} & = & 0 & & \\ (x^2+y^2) & = & C & & \end{array}\right)$$

$$[\mathrm{E}_2][\mathrm{S}_2] = \left(\begin{array}{rcrcl} (2x) + (2y)\frac{dy}{dx} & = & 0 & & \\ (x^2+y^2) & = & C & & \end{array}\right)$$

Uma questão da P2 de 2019.2

4) Sejam (∗∗∗) e (∗∗∗∗) estas EDOs:

$$2xy^3\,dx + 3x^2y^2\,dy = 0 \quad (***)$$
$$2x^2y^3\,dx + 3x^3y^2\,dy = 0 \quad (****)$$

a) **(0.5 pts)** Mostre que (∗∗∗) é exata.
b) **(0.5 pts)** Encontre a solução geral de (∗∗∗).
c) **(1.0 pts)** Teste a sua solução geral da (∗∗∗).
d) **(0.5 pts)** Mostre que a solução geral da EDO (∗∗∗) também é solução da (∗∗∗∗).
e) **(0.5 pts)** Mostre que (∗∗∗∗) não é exata.
f) **(0.5 pts)** Mostre que o fator integrante obtido por

$$\begin{aligned} p(x) &= (M_y - N_x)/N, \\ \mu(x) &= e^{\int p(x)\,dx} \end{aligned}$$

transforma (∗∗∗∗) em (∗∗∗).

Versão original:
http://angg.twu.net/LATEX/2019-2-C2-P2.pdf

```
(%i1) load("2025-1-exact.mac")$
(%i2) ee : Exact_from0()$
(%i3) ee@@E5();
```
(%o3)
$$\begin{pmatrix} d\,z & = & z_x\,\mathrm{dx} + z_y\,\mathrm{dy} & = & 0 \\ \frac{d}{d\,x}\,z & = & z_x + z_y\,\left(\frac{d}{d\,x}\,y\right) & = & 0 \\ z & = & C & & \end{pmatrix}$$
```
(%i4) ee : Exact_from1(x^2+y^2)$
(%i5) ee@@E5();
```
(%o5)
$$\begin{pmatrix} d\,\left(y^2+x^2\right) & = & 2\,x\,\mathrm{dx} + 2\,y\,\mathrm{dy} & = & 0 \\ \frac{d}{d\,x}\,\left(y^2+x^2\right) & = & 2\,x + 2\,y\,\left(\frac{d}{d\,x}\,y\right) & = & 0 \\ y^2+x^2 & = & C & & \end{pmatrix}$$
```
(%i6) ee : Exact_from2(2*x, 2*y)$
(%i7) ee@@E5();
```
(%o7)
$$\begin{pmatrix} d\,z & = & 2\,x\,\mathrm{dx} + 2\,y\,\mathrm{dy} & = & 0 \\ \frac{d}{d\,x}\,z & = & 2\,x + 2\,y\,\left(\frac{d}{d\,x}\,y\right) & = & 0 \\ z & = & C & & \end{pmatrix}$$
```
(%i8) ee@@ode();
```
(%o8)
$$2\,y\,\left(\frac{d}{d\,x}\,y\right) + 2\,x = 0$$
```
(%i9) ee@@odesolve();
```
(%o9)
$$-\left(\frac{y^2}{2}\right) = \frac{x^2}{2} + \%\mathrm{c}$$
```
(%i10) sols  : ee@@odesols();
```
(%o10)
$$\left[y = -\sqrt{-x^2 - 2\,\%\mathrm{c}}, y = \sqrt{-x^2 - 2\,\%\mathrm{c}}\right]$$
```
(%i11) sol   : sols[1];
```
(%o11)
$$y = -\sqrt{-x^2 - 2\,\%\mathrm{c}}$$
```
(%i12) test1 : ee@@odesubst(sol);
```
(%o12)
$$2\,x - 2\,\sqrt{-x^2 - 2\,\%\mathrm{c}}\,\left(\frac{d}{d\,x}\,\left(-\sqrt{-x^2 - 2\,\%\mathrm{c}}\right)\right) = 0$$
```
(%i13) test2 : ev(test1, diff);
```
(%o13)
$$0 = 0$$

```
(%i1) load("2025-1-exact.mac")$
(%i2) ee : Exact_from1(x^2*y^3);
(%o2)
```

$$\text{Exact}\left(x^2\,y^3, 2\,x\,y^3, 3\,x^2\,y^2\right)$$

```
(%i3) ee@@caixinhas_3();
(%o3)
```

$$\left[\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\right]$$

```
(%i4) ee@@caixinhas_2();
(%o4)
```

$$\left[\begin{pmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\right]$$

```
(%i5) ee : Exact_from1(x^2+y^2);
(%o5)
```

$$\text{Exact}\left(y^2 + x^2, 2\,x, 2\,y\right)$$

```
(%i6) ee@@caixinhas_3();
(%o6)
```

$$\left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\right]$$

```
(%i7) ee@@caixinhas_2();
(%o7)
```

$$\left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\right]$$

```
(%i1) [maxxp,maxyp] : [2,2];
```
(%o1)
$$[2,2]$$

```
(%i2) "1"$  ee : Exact_from3("?", 2*x+3, 2*y-2);
```
(%o3)
$$\text{Exact}\left(\ ?\ , 2\,x+3, 2\,y-2\right)$$

```
(%i4) ee@@caixinhas_2();
```
(%o4)
$$\left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}\right]$$

```
(%i5) ee@@effsc();
```
(%o5)
$$\left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} ? & 0 & 0 \\ ? & 0 & 0 \\ ? & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ -2 & 0 & 0 \\ ? & ? & ? \end{pmatrix}\right]$$

```
(%i6) "2"$  ee : Exact_from3("?", 2*x+4*y, 2*x-2*y);
```
(%o7)
$$\text{Exact}\left(\ ?\ , 4\,y+2\,x, 2\,x-2\,y\right)$$

```
(%i8) ee@@caixinhas_2();
```
(%o8)
$$\left[\begin{pmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ -2 & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix}\right]$$

```
(%i9) ee@@effsc();
```
(%o9)
$$\left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 4 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} ? & 0 & 0 \\ ? & 4 & 0 \\ ? & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ ? & ? & ? \end{pmatrix}\right]$$

```
(%i10) [maxxp,maxyp] : [3,3];
```
(%o10)
$$[3,3]$$

```
(%i11) "3"$  ee : Exact_from3("?", 3*x^2 - 2*x*y + 2, 6*y^2 - x^2 + 3);
```
(%o12)
$$\text{Exact}\left(\ ?\ , -\left(2\,x\,y\right)+3\,x^2+2, 6\,y^2-x^2+3\right)$$

```
(%i13) ee@@caixinhas_2();
```
(%o13)
$$\left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 2 & 0 & 3 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & 0 & -1 & 0 \end{pmatrix}\right]$$

```
(%i14) ee@@effsc();
```
(%o14)
$$\left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} ? & 0 & 0 & 0 \\ ? & 0 & 0 & 0 \\ ? & 0 & -1 & 0 \\ ? & 2 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ ? & ? & ? & ? \end{pmatrix}\right]$$