On a way to visualize some Grothendieck Topologies

Eduardo Ochs*
UFF, Rio das Ostras, RJ, Brazil

Abstract

The *canonical Grothendieck topology* on $\mathbb{R}$, J_{can}, is easy to define, but the definition takes several steps: 1) for each open set $U \in \mathcal{O}(\mathbb{R})$ a *sieve on* U is a subset of $\mathcal{O}(U)$ that is downward-closed; 2) for each $U \in \mathcal{O}(\mathbb{R})$ we write $\Omega(U)$ for the set of all sieves on U; 3) we say that a sieve $\mathcal{S} \in \Omega(U)$ is *covering* when $\bigcup \mathcal{S} = U$; 4) for each $U \in \mathcal{O}(\mathbb{R})$ we define $J_{\mathrm{can}}(U)$ as the set of covering sieves on U.

The "real" definition of Grothendieck Topology generalizes this definition of J_{can} in many ways: in particular, it starts with a category $\mathbf{C}$ instead of a topological space $(\mathbb{R}, \mathcal{O}(\mathbb{R}))$, and we can have many notions of "covering-ness" for the same category — they just have to obey the three axioms in [1], p.110.

In this presentation I will show how we can use some of the techniques in [2] to understand the general definition of Grothendieck Topology, and I will show how we can visualize all the Grothendieck Topologies on one of the Planar Heyting Algebras of [4]. Most of the diagrams in the presentation will be taken from [3].

References

[1] Mac Lane, S.; Moerdijk, I. *Sheaves in geometry and logic: a first introduction to topos theory.* Springer, 1992.

[2] Ochs, E. On the missing diagrams in Category Theory. In: L. Magnani (editor), *Handbook of Abductive Reasoning*. Springer, 2022. Available at `http://angg.twu.net/math-b.html#2022-md`.

[3] Ochs, E. *Grothendieck Topologies for Children.* Available at `http://angg.twu.net/math-b.html#2021-groth-tops`.

[4] Ochs, E. Planar Heyting Algebras for Children. *South American Journal of Logic* 5.1:125–164, 2019. Available at `http://angg.twu.net/math-b.html#zhas-for-children-2`.

*eduardoochs@gmail.com