Notes on Bart Jacobs's book
"Categorical Logic and Type Theory" (Elsevier, 1999).
http://www.cs.ru.nl/B.Jacobs/CLT/bookinfo.html
http://www.math.mcgill.ca/rags/jacobs.html
These notes are at:
http://angg.twu.net/LATEX/2020jacobs.pdf
See:
http://angg.twu.net/LATEX/2020favorite-conventions.pdf
http://angg.twu.net/math-b.html#favorite-conventions
I wrote these notes mostly to test if the conventions above are good enough.

4.8. Quotient types, categorically

(Page 291):

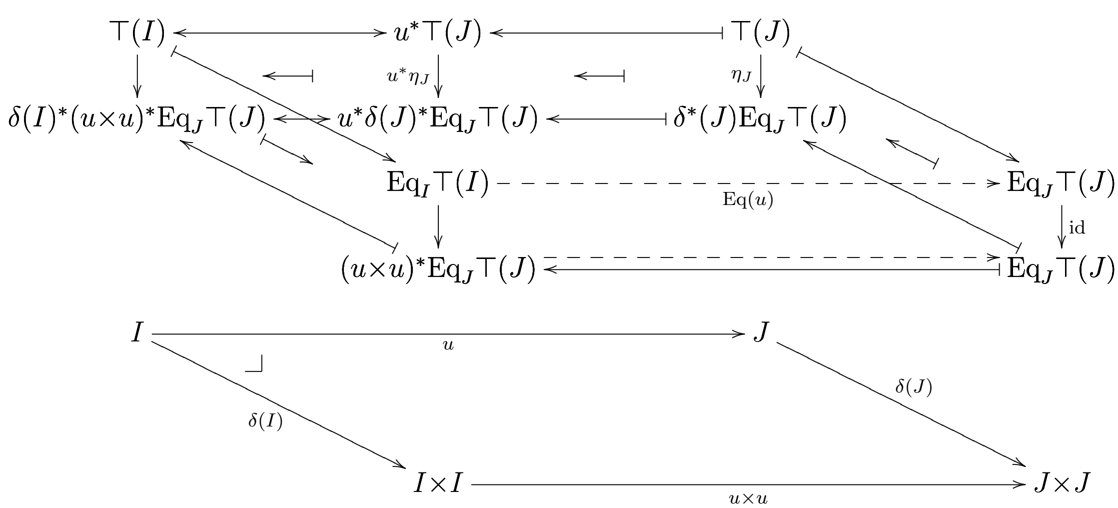
A morphism $u : I \to J$ in $\mathbb{B}$ is mapped to the composite

$$\mathrm{Eq}(I) = \mathrm{Eq}_I \top(I) \longrightarrow (u \times u)^* \mathrm{Eq}_J \top(J) \longrightarrow \mathrm{Eq}_J \top(J) = \mathrm{Eq}(J)$$

where the first part of this map is obtained by transposing the following composite across the adjunction $\mathrm{Eq}_I \dashv \delta(I)^*$:

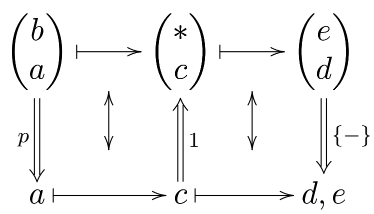
$$\top(I) \cong u^* \top(J) \xrightarrow{u^* \eta_J} u^* \delta(J)^* \mathrm{Eq}_J \top(J) \cong \delta(I)^* (u \times u)^* \mathrm{Eq}_J \top(J)$$

Here is a diagram that explains that construction. Two morphisms in the construction above are morphisms in $\mathbb{E}$ that are not-vertical; I drew them as '$- - \rightarrow$'s.

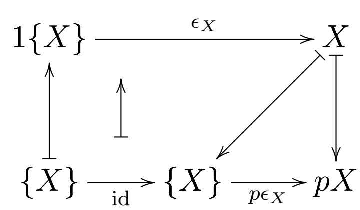


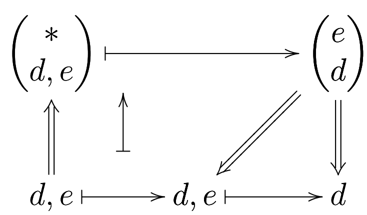
From this point on all the diagrams are from notes that I wrote in 2008, using almost only the "downcased notation" from [IDARCT, sec.3]... *TODO:* create diagrams similar to these using the notation from Jacobs! See [FavC], section 2, conventions (COT) and (CNSh).

Comprehension categories with unit
Jacobs, 10.4.7 (p.616):
A fibration $p : \mathbb{E} \to \mathbb{B}$ with a terminal object functor $1 : \mathbb{B} \to \mathbb{E}$
(where we know by lemma 1.8.8 that $p \dashv 1$ and that $\eta_I = \mathrm{id}$)
is *comprehension category with unit* if 1 has a right adjoint.
We call this right adjoint $\{-\}$.



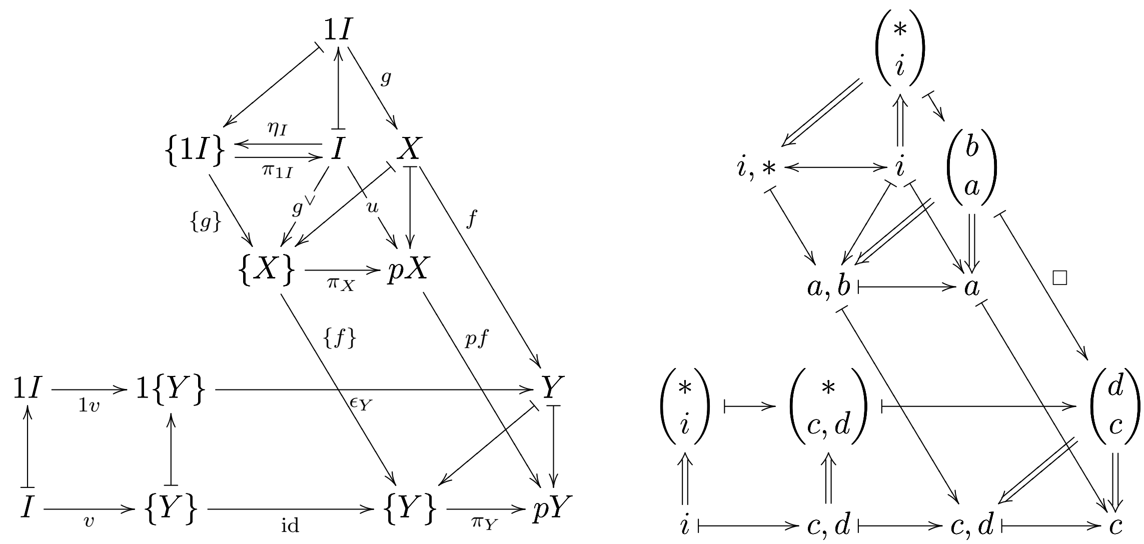
Jacobs, 10.4.7 (p.616):
Definition of the functor $\mathbb{E} \to \mathbb{B}^{\to}$:
its action on objects is $X \mapsto p\epsilon_X$.



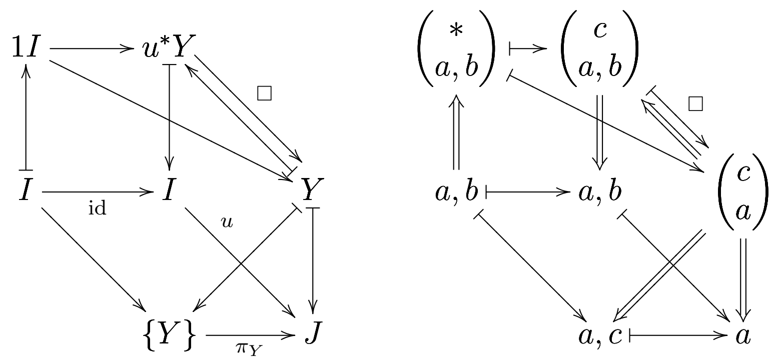


The functor $\mathbb{E} \to \mathbb{B}^{\to}$ is a comprehension category, i.e.,
it takes cartesian morphisms to pullback squares.
We want to check that the image of a cartesian morphism is a pullback.
Given two maps $i \mapsto a$ and $i \mapsto c, d$ such that
$a \mapsto c$ is well-defined, we need to construct a

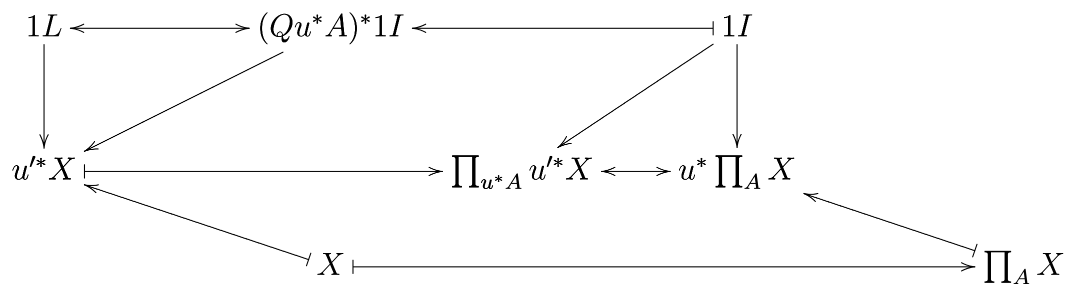
mediating map $i \mapsto a, b$.

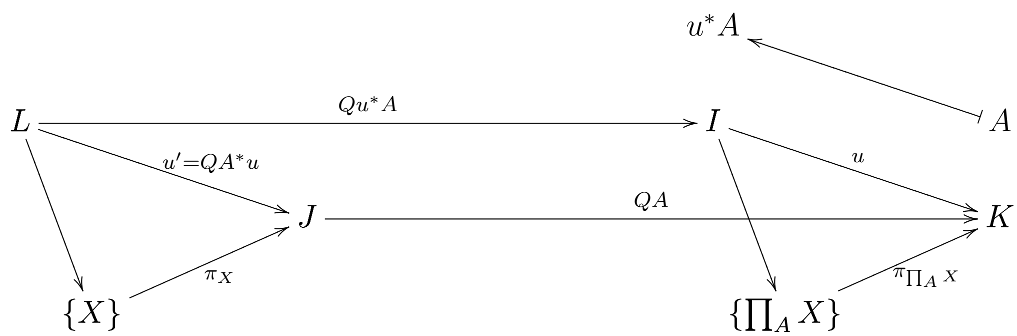


Comprehension categories with unit: a bijection
Jacobs, 10.4.9 (i):
In a CCw1, pack a morphism $u : I \to J$ in the base category, and an object Y over J. Then the vertical morphisms $1I \to u^*Y$ are in bijection with morphisms from u to π_Y in $\mathbb{B}/J$.

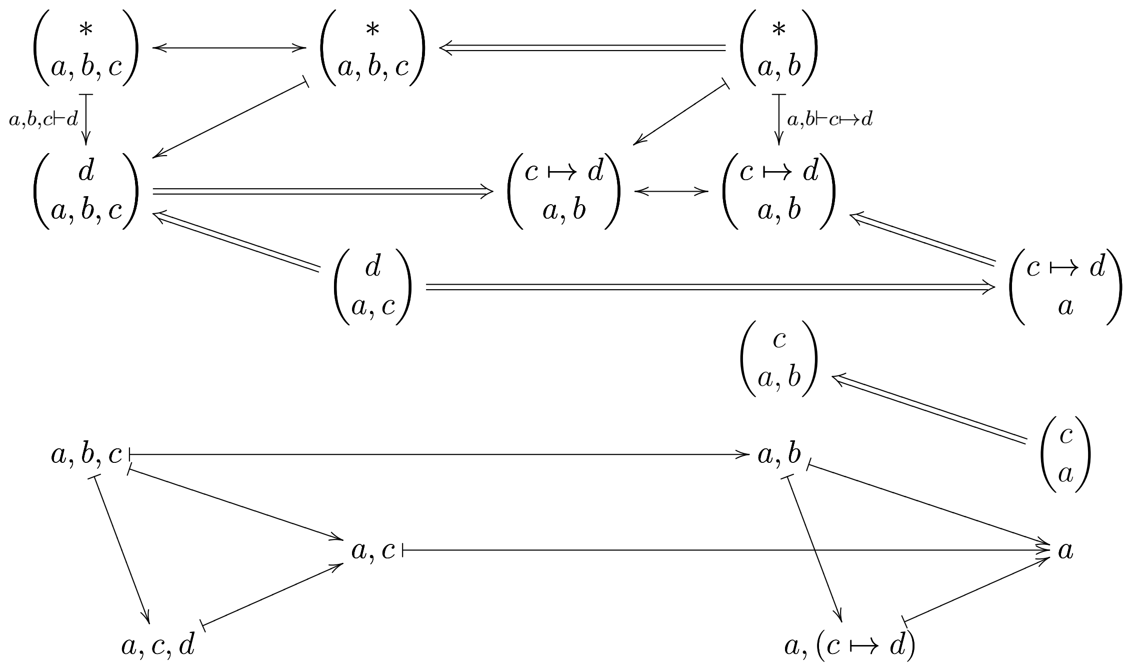


Comprehension categories with unit: big bijection
Jacobs, 10.4.9 (ii):



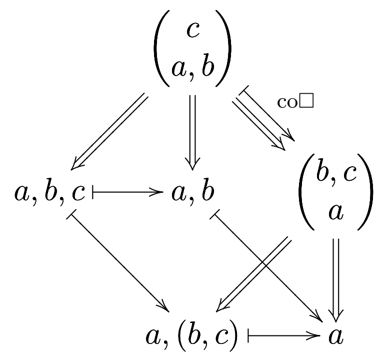


$a, c; b \vdash d$
$a; b \vdash c \mapsto d$

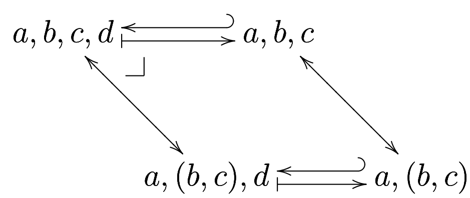

a,b,c⊢d
a,b⊢c↦d

Comprehension categories with unit: three rules
Jacobs, 10.3.3:
The categorical interpretation of
the rules for dependent sums:

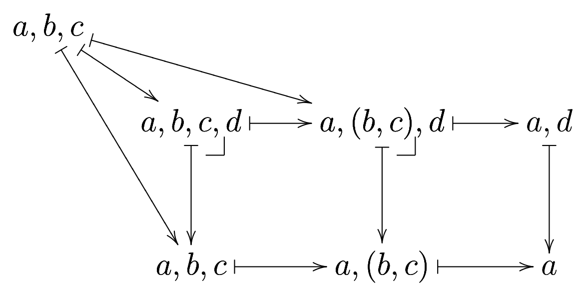
$$\frac{a, b \vdash C}{a; b, c \vdash (b, c)}\ \Sigma\mathrm{I}$$



$$\frac{a, (b, c) \vdash D \quad a, b, c \vdash d}{a, (b, c) \vdash d}\ \Sigma\mathrm{E}^{+}$$

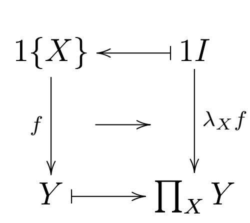


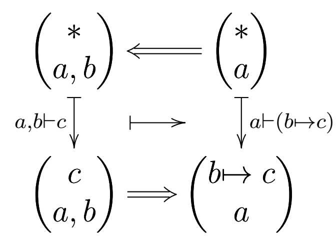
$$\frac{a \vdash D \quad a, b, c \vdash d}{a, (b, c) \vdash d}\ \Sigma\mathrm{E}^{-}$$

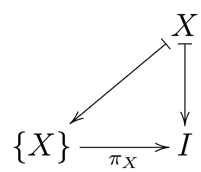


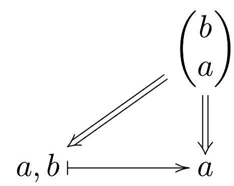
(Oops, the diagram for $\Sigma\mathrm{E}^{-}$ is wrong)

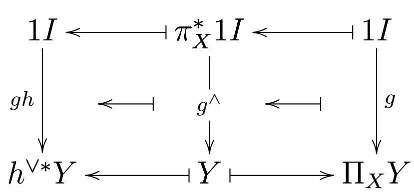
Interpreting ΠI **and** ΠE **in a CCompC**
(Jacobs, 10.5.3)

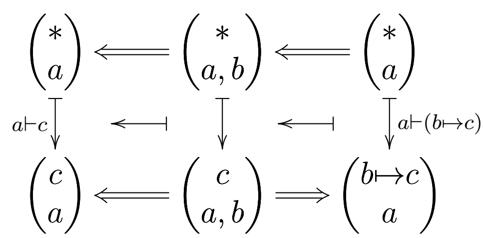


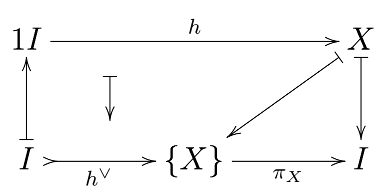


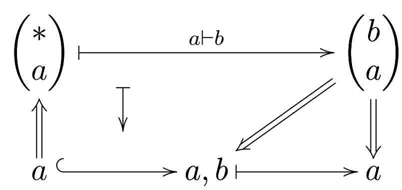






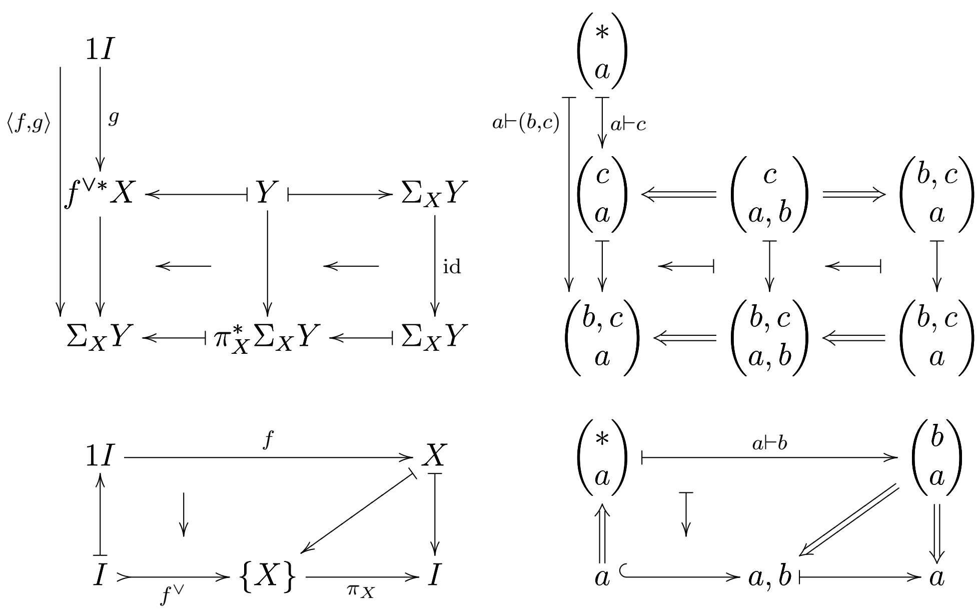




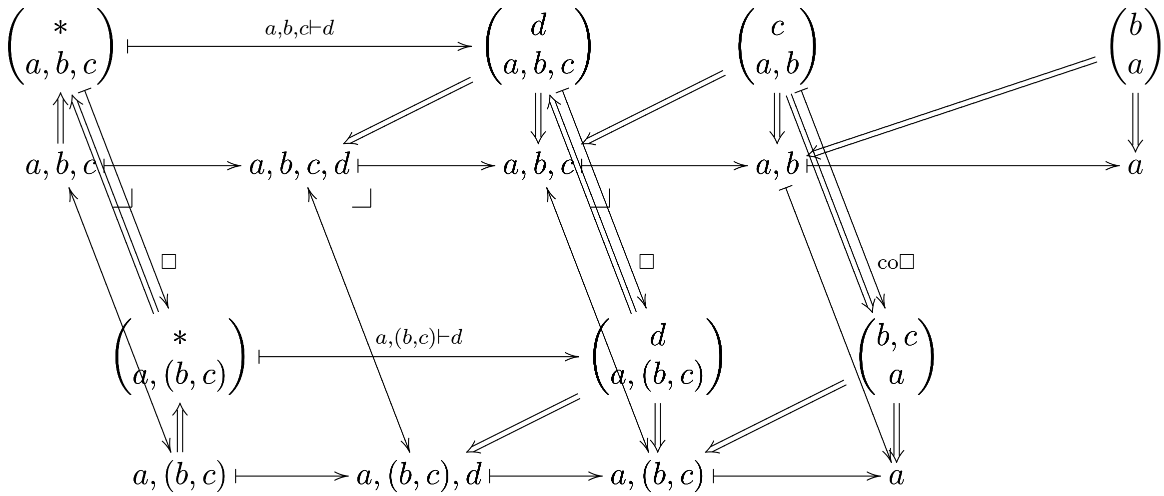


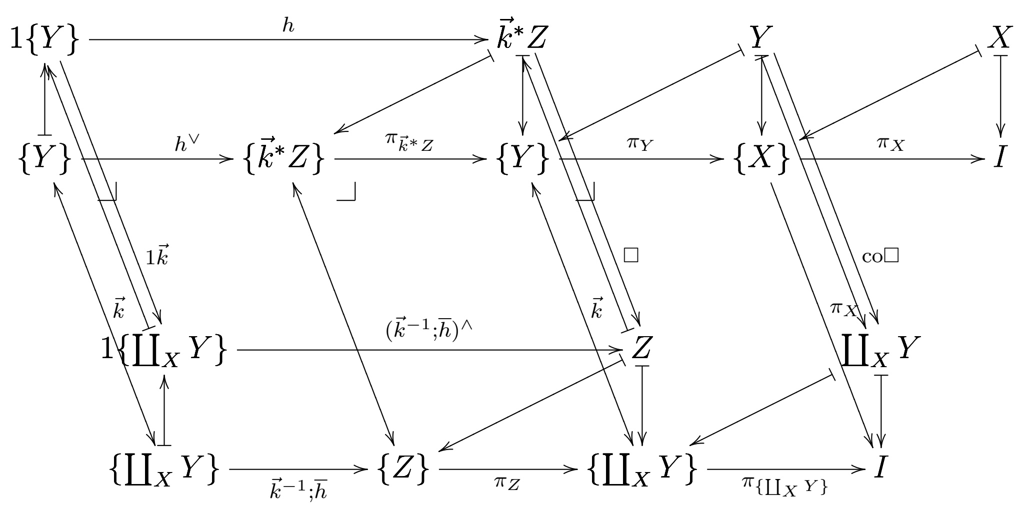
In the top left vertex of the diagram for ΠE we have omitted an iso to keep the diagram shorter: $1I \cong h^{\vee *}\pi_X^* 1I$.

Interpreting ΣI in a CCompC
(Jacobs, 10.5.3)



Interpreting ΣE^+ in a CCompC
(Jacobs, 10.5.3)





References

[FavC] E. Ochs. "On my favorite conventions for drawing the missing diagrams in Category Theory". http://angg.twu.net/math-b.html#favorite-conventions. 2020.

[IDARCT] E. Ochs. "Internal Diagrams and Archetypal Reasoning in Category Theory". In: *Logica Universalis* 7.3 (Sept. 2013). http://angg.twu.net/math-b.html#idarct, pp. 291–321.