

Cálculo 2  
 PURO-UFF - 2019.1  
 P1 - turma pequena - 30/outubro/2019 - Eduardo Ochs  
 Respostas sem justificativas não serão aceitas.  
 Proibido usar quaisquer aparelhos eletrônicos.

1) **(Total: 2.5)** Calcule

$$\int (2x + 3)\sqrt{4x + 5} dx.$$

2) **(Total: 2.5)** Calcule

$$\int x^3 \sqrt{1 - x^2} dx.$$

3) **(Total: 2.5)** Calcule

$$\int (\sin 5x)^2 (\cos 6x)^2 dx.$$

4) **(Total: 2.5)** Calcule

$$\int \frac{x^3}{x^2 + 9x + 20} dx.$$

Algumas definições, fórmulas e substituições:

$$\begin{array}{llll} c = \cos \theta & c^2 + s^2 = 1 & \frac{ds}{d\theta} = c & E = c + is \\ s = \sin \theta & z^2 = t^2 + 1 & \frac{dc}{d\theta} = -s & c = \frac{E+E^{-1}}{2} \\ t = \tan \theta & \sqrt{1-s^2} = c & \frac{dt}{d\theta} = z^2 & s = \frac{E-E^{-1}}{2i} \\ z = \sec \theta & \sqrt{t^2+1} = z & \frac{dz}{d\theta} = zt & e^{ik\theta} + e^{-ik\theta} = 2 \cos k\theta \\ E = e^{i\theta} & \sqrt{z^2-1} = t & & e^{ik\theta} - e^{-ik\theta} = 2i \sin k\theta \end{array}$$

Cálculo 2  
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 P1 - turma grande - 31/outubro/2019 - Eduardo Ochs  
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Cálculo 2  
 PURO-UFF - 2019.1  
 P1 - versão pra Thais Knupp - 6/novembro/2019 - Eduardo Ochs  
 Respostas sem justificativas não serão aceitas.  
 Proibido usar quaisquer aparelhos eletrônicos.

1) **(Total: 2.5)** Calcule

$$\int (2 + 3x)\sqrt{4 + 5x} dx.$$

2) **(Total: 2.5)** Calcule

$$\int x^3 \sqrt{1 - x^2} dx.$$

3) **(Total: 2.5)** Calcule

$$\int (\sin 4x)^2 (\cos 5x)^2 dx.$$

4) **(Total: 2.5)** Calcule

$$\int \frac{x^3}{x^2 + 7x + 12} dx.$$

Algumas definições, fórmulas e substituições:

$$\begin{array}{llll} c = \cos \theta & c^2 + s^2 = 1 & \frac{ds}{d\theta} = c & E = c + is \\ s = \sin \theta & z^2 = t^2 + 1 & \frac{dc}{d\theta} = -s & c = \frac{E+E^{-1}}{2} \\ t = \tan \theta & \sqrt{1-s^2} = c & \frac{dt}{d\theta} = z^2 & s = \frac{E-E^{-1}}{2i} \\ z = \sec \theta & \sqrt{t^2+1} = z & \frac{dz}{d\theta} = zt & e^{ik\theta} + e^{-ik\theta} = 2 \cos k\theta \\ E = e^{i\theta} & \sqrt{z^2-1} = t & & e^{ik\theta} - e^{-ik\theta} = 2i \sin k\theta \end{array}$$

Gabarito (incompleto e não revisado):

$$\begin{aligned}
 1) \int (2+3x)\sqrt{4+5x} dx &= \int (2+3(\frac{u}{5}-\frac{4}{5}))\sqrt{u}(\frac{1}{5}) du && \begin{cases} u=4+5x \\ x=\frac{u-4}{5} \\ dx=\frac{1}{5}du \end{cases} \\
 &= \int (\frac{1}{5})(\frac{3}{5}u + \frac{10}{5} - \frac{12}{5})\sqrt{u} du \\
 &= \int (\frac{3}{25}u - \frac{2}{25})\sqrt{u} du \\
 &= \int \frac{3}{25}u^{3/2} - \frac{2}{25}u^{1/2} du \\
 &= \frac{3}{25} \frac{5}{2} u^{5/2} - \frac{2}{25} \frac{2}{3} u^{3/2} \\
 &= \frac{3}{10} u^{5/2} - \frac{4}{75} u^{3/2} \\
 &= \frac{3}{10} (4+5x)^{5/2} - \frac{4}{75} (4+5x)^{3/2}
 \end{aligned}$$

$$\begin{aligned}
 2) \int x^3 \sqrt{1-x^2} dx & \quad \begin{cases} s=x \\ s=\text{sen } \theta \\ ds=\cos \theta d\theta \\ \sqrt{1-s^2}=\cos \theta \end{cases} \\
 &= \int s^3 \sqrt{1-s^2} ds \\
 &= \int (\text{sen } \theta)^3 \cos \theta \cos \theta ds \\
 &= \int (\text{sen } \theta)^2 (\cos \theta)^2 \text{sen } \theta ds && \begin{cases} \cos \theta = c \\ (\text{sen } \theta)^2 = 1 - c^2 \\ \text{sen } \theta d\theta = (-1)dc \end{cases} \\
 &= \int (1-c^2)c^2(-1)dc \\
 &= \int c^4 - c^2 dc \\
 &= \frac{1}{5}c^5 - \frac{1}{3}c^3 \\
 &= \frac{1}{5}\sqrt{1-s^2}^5 - \frac{1}{3}\sqrt{1-s^2}^3 \\
 &= \frac{1}{5}\sqrt{1-x^2}^5 - \frac{1}{3}\sqrt{1-x^2}^3
 \end{aligned}$$

$$\begin{aligned}
 3) (\text{sen } 5\theta)^2 (\cos 6\theta)^2 &= \left( \frac{E^5 - E^{-5}}{2i} \right)^2 \left( \frac{E^6 - E^{-6}}{2} \right)^2 \\
 &= -\frac{1}{16} (E^{10} - 2 + E^{-10}) (E^{12} - 2 + E^{-12}) \\
 &= -\frac{1}{16} \begin{pmatrix} (E^{10} - 2 + E^{-10})E^{12} + \\ (E^{10} - 2 + E^{-10})(-2) + \\ (E^{10} - 2 + E^{-10})E^{-12} \\ E^{22} - 2E^{12} + E^2 + \\ -2E^{10} + 4 - 2E^{-10} + \\ E^{-2} - 2E^{-12} + E^{-22} \end{pmatrix} \\
 &= -\frac{1}{16} ((E^{22} + E^{-22}) - 2(E^{12} + E^{-12}) - 2(E^{10} + E^{-10}) + (E^2 + E^{-2}) + 4) \\
 &= -\frac{1}{16} (2 \cos 22\theta - 4 \cos 12\theta - 4 \cos 10\theta + 2 \cos 2\theta + 4) \\
 &= -\frac{1}{8} \cos 22\theta + \frac{1}{4} \cos 12\theta + \frac{1}{4} \cos 10\theta - \frac{1}{8} \cos 2\theta
 \end{aligned}$$

$$\begin{aligned}
 \int (\text{sen } 5x)^2 (\cos 6x)^2 dx &= \int -\frac{1}{8} \cos 22x + \frac{1}{4} \cos 12x + \frac{1}{4} \cos 10x - \frac{1}{8} \cos 2x \frac{1}{4} dx \\
 &= -\frac{1}{8 \cdot 22} \text{sen } 22x + \frac{1}{4 \cdot 12} \text{sen } 12x + \frac{1}{4 \cdot 10} \text{sen } 10x - \frac{1}{8 \cdot 2} \text{sen } 2x \frac{1}{4} x
 \end{aligned}$$

$$\begin{aligned} 4) \int \frac{x^3}{x^2 + 7x + 12} dx &= \int x - 7 + \frac{37x + 84}{x^2 + 7x + 12} dx \\ &= \int x - 7 + \frac{37x + 84}{(x + 3)(x + 4)} dx \\ &= \int x - 7 - \frac{27}{x + 3} + \frac{64}{x + 4} dx \\ &= \frac{x^2}{2} - 7x + 27 \ln |x + 3| + 64 \ln |x + 4| \end{aligned}$$