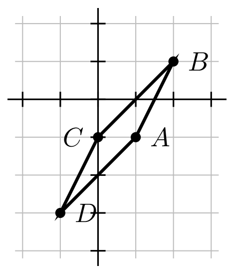
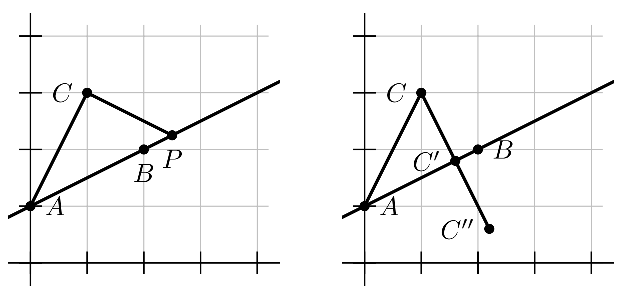
Geometria Analítica
PURO-UFF - 2015.2
Mini-gabarito da P1 - Eduardo Ochs
Links importantes:
http://angg.twu.net/2015.2-GA.html (página do curso)
http://angg.twu.net/2015.2-GA/2015.2-GA.pdf (quadros)
http://angg.twu.net/LATEX/2015-2-GA-P1-gab.pdf
eduardoochs@gmail.com (meu e-mail)

1) 


$\overrightarrow{AB} = \overrightarrow{(1,2)} = \overrightarrow{DC}$
$\overrightarrow{BC} = \overrightarrow{(-2,-2)} = \overrightarrow{AD}$
Área $= \begin{vmatrix} 1 & -2 \\ 2 & -2 \end{vmatrix} = 2$

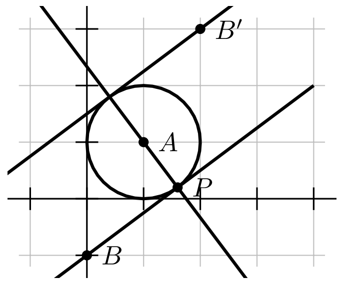
2) 


2a) $l = \{\, (0,1) + t\overrightarrow{(2,1)} \mid t \in \mathbb{R} \,\} = \{\, (x,y) \in \mathbb{R}^2 \mid y = 1 + \frac{x}{2} \,\}$

2b) $r = \{\, A + t\overrightarrow{AC} \mid t \in \mathbb{R} \,\} = \{\, (x,y) \in \mathbb{R}^2 \mid y = 1 + 2x \,\}$
$s = \{\, C + t\overrightarrow{(2,1)} \mid t \in \mathbb{R} \,\} = \{\, (x,y) \in \mathbb{R}^2 \mid y = 3.5 - \frac{x}{2} \,\}$
$P = (x,y) \in l \cap s$
$1 + \frac{x}{2} = 3.5 - \frac{x}{2} \quad \Rightarrow \quad x = 2.5$
$y = 1 + \frac{x}{2} = 1 + \frac{2.5}{2} = 2.25 \quad \Rightarrow \quad P = (2.5, 2.25)$

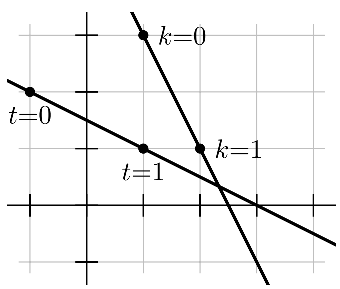
2c) $\mathrm{Pr}_{\overrightarrow{AB}} \overrightarrow{PC} = \mathrm{Pr}_{\overrightarrow{(2,1)}} \overrightarrow{(-1.5, 0.75)} = \frac{-3+0.75}{5}\overrightarrow{(2,1)} = -0.45\overrightarrow{(2,1)} = \overrightarrow{(-0.9, -0.45)}$

2d) $\mathrm{Pr}_{\overrightarrow{AB}} \overrightarrow{AC} = \mathrm{Pr}_{\overrightarrow{(2,1)}} \overrightarrow{(1,2)} = \frac{4}{5}\overrightarrow{(2,1)} = \overrightarrow{(1.6, 0.8)}$
$C' := A + \mathrm{Pr}_{\overrightarrow{AB}} \overrightarrow{AC} = (0,1) + \overrightarrow{(1.6, 0.8)} = (1.6, 1.8)$
$\overrightarrow{CC'} = C' - C = (1.6, 1.8) - (1,3) = \overrightarrow{(0.6, -1.2)}$
$C'' := C' + \overrightarrow{C'C''} = C' + \overrightarrow{CC'} = (1,3) + \overrightarrow{(0.6, -1.2)} = (2.2, 0.6)$

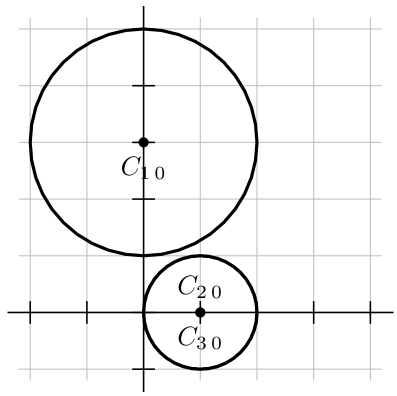
3) 


a) $r = \{\, (x,y) \in \mathbb{R}^2 \mid 3x - 4y - 4 = 0 \,\} = \{\, (x,y) \in \mathbb{R}^2 \mid y = \frac{3}{4}x - 1 \,\}$
$d((1,1), r) = \frac{5/4}{\sqrt{1+\frac{9}{16}}} = \frac{5/4}{\sqrt{25/16}} = \frac{5/4}{5/4} = 1$
$C = \{\, (x,y) \in \mathbb{R}^2 \mid (x-1)^2 + (y-1)^2 = 1 \,\}$

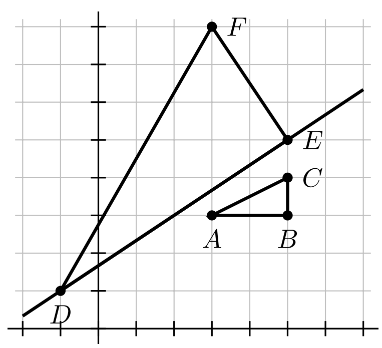
b) $B' := A + 2\overrightarrow{BA} = (0,-1) + 2\overrightarrow{(1,2)} = (2,3)$
$s = \{\, (2,3) + t\overrightarrow{(1, \frac{3}{4})} \mid t \in \mathbb{R} \,\} = \{\, (x,y) \in \mathbb{R}^2 \mid y = \frac{3}{4}x + 1.5 \,\}$

4) 


$r = \{\, (-1,2) + t\overrightarrow{(2,-1)} \mid t \in \mathbb{R} \,\} = \{\, (x,y) \in \mathbb{R}^2 \mid y = 1.5 - \frac{x}{2} \,\}$
$s = \{\, (1,3) + k\overrightarrow{(1,-2)} \mid k \in \mathbb{R} \,\} = \{\, (x,y) \in \mathbb{R}^2 \mid y = 5 - 2x \,\}$
$P = (x,y) \in r \cap s$
$1.5 - \frac{x}{2} = 5 - 2x \quad \Rightarrow \quad 1.5x = 3.5 \quad \Rightarrow \quad x = \frac{7}{3}$
$y = 5 - 2\frac{7}{3} = \frac{15}{3} - \frac{14}{3} = \frac{1}{3} \quad \Rightarrow \quad P = (\frac{7}{3}, \frac{1}{3})$
Sejam $\vec{u} := \overrightarrow{(2,-1)}$ e $\vec{v} := \overrightarrow{(1,-2)}$. Temos $\|\vec{u}\| = \|\vec{v}\|$, então
$b = \{\, P + t(\vec{u} + \vec{v}) \mid t \in \mathbb{R} \,\} = \{\, (\frac{7}{3}, \frac{1}{3}) + t(3,-3) \mid t \in \mathbb{R} \,\}$ e
$b' = \{\, P + t(\vec{u} - \vec{v}) \mid t \in \mathbb{R} \,\} = \{\, (\frac{7}{3}, \frac{1}{3}) + t(1,1) \mid t \in \mathbb{R} \,\}$
são bissetrizes de $r$ e $s$.

5) 


$$\begin{aligned}
C_1 &= \{\, (0,3) + 2\overrightarrow{(\cos\theta, \operatorname{sen}\theta)} \mid \theta \in \mathbb{R} \,\} \\
&= \{\, (x,y) \in \mathbb{R}^2 \mid x^2 + (y-3)^2 = 2^2 \,\} \\
C_2 &= \{\, (x,y) \in \mathbb{R}^2 \mid x^2 - 2x + y^2 = 0 \,\} \\
&= \{\, (x,y) \in \mathbb{R}^2 \mid (x-1)^2 - 1 + y^2 = 0 \,\} \\
&= \{\, (x,y) \in \mathbb{R}^2 \mid (x-1)^2 + y^2 = 1 \,\} \\
C_3 &= \{\, (x,y) \in \mathbb{R}^2 \mid (x-1)^2 + y^2 = 1 \,\}
\end{aligned}$$

6) 


$r = \{\, (x,y) \in \mathbb{R}^2 \mid 2x - 3y + 5 = 0 \,\} = \{\, D + t\overrightarrow{(3,2)} \mid t \in \mathbb{R} \,\}$
$E := D + 2\overrightarrow{(3,2)}$
$F := D + 2\overrightarrow{(3,2)} + \overrightarrow{(-2,3)}$