Cálculo 2
PURO-UFF - 2015.2
P1 - 14/mar/2016 - Eduardo Ochs
Links importantes:
http://angg.twu.net/2015.2-C2.html (página do curso)
http://angg.twu.net/2015.2-C2/2015.2-C2.pdf (quadros)
http://angg.twu.net/LATEX/2015-2-C2-material.pdf
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1) Seja $f_a(x) = x^2 \cos ax$. Calcule:

$$a)\ \ (2.0 \text{ pts}) \qquad \int f_a(x)\,dx$$

$$b)\ \ (1.0 \text{ pts}) \qquad \int_{x=0}^{x=\pi} f_2(x)\,dx$$

2) Calcule:

$$(3.0 \text{ pts}) \qquad \int \frac{x^3}{x^2+x-20}\,dx$$

3) Calcule:

$$a)\ \ (2.0 \text{ pts}) \qquad \int \frac{\sqrt{x^2-1}}{x^3}\,dx$$

$$b)\ \ (2.0 \text{ pts}) \qquad \int \frac{\sqrt{4x^2-9}}{x^3}\,dx$$

Método de Heaviside:
Se $f(x) = \frac{\alpha}{x-a} + \frac{\beta}{x-b} + \frac{\gamma}{x-c} = \frac{p(x)}{(x-a)(x-b)(x-c)}$,
então $\lim_{x\to a} f(x)(x-a) = \alpha = \frac{p(a)}{(a-b)(a-c)}$.

Substituição:

$$\begin{array}{l} g(h(x))|_{x=a}^{x=b} = \int_{x=a}^{x=b} g'(h(x))\frac{d\,h(x)}{dx}\,dx \\ \quad || \\ g(u))|_{u=h(a)}^{u=h(b)} = \int_{u=h(a)}^{u=h(b)} g'(u)\,du \end{array}$$

Fórmulas:

$$\begin{array}{lll} \int_{x=a}^{x=b} f(g(x))\frac{d\,g(x)}{dx}\,dx & \int f(g(x))\frac{d\,g(x)}{dx}\,dx & \\ = \int_{x=a}^{x=b} f(u)\frac{du}{dx}\,dx & = \int f(u)\frac{du}{dx}\,dx & \left[\, u=g(x) \,\right] \\ = \int_{u=g(a)}^{u=g(b)} f(u)\,du & = \int f(u)\,du & \left[\, u=g(x) \,\right] \end{array}$$

Substituição inversa:

$$\begin{array}{l} g(h(x))|_{x=h^{-1}(\alpha)}^{x=h^{-1}(\beta)} = \int_{x=h^{-1}(\alpha)}^{x=h^{-1}(\beta)} g'(h(x))\frac{d\,h(x)}{dx}\,dx \\ \quad || \\ g(u))|_{u=h(h^{-1}(\alpha))}^{u=h(h^{-1}(\beta))} = \int_{u=h(h^{-1}(\alpha))}^{u=h(h^{-1}(\beta))} g'(u)\,du \\ \quad || \\ g(u))|_{u=\alpha}^{u=\beta} = \int_{u=\alpha}^{u=\beta} g'(u)\,du \end{array}$$

Fórmulas:

$$\begin{array}{lll} \int_{u=\alpha}^{u=\beta} f(u)\,du & \int f(u)\,du & \\ = \int_{x=g^{-1}(\alpha)}^{x=g^{-1}(\beta)} f(u)\frac{du}{dx}\,dx & = \int f(u)\frac{du}{dx}\,dx & \left[\begin{array}{c} u=g(x) \\ x=g^{-1}(u) \end{array}\right] \\ = \int_{x=g^{-1}(\alpha)}^{x=g^{-1}(\beta)} f(g(x))\frac{d\,g(x)}{dx}\,dx & = \int f(g(x))\frac{d\,g(x)}{dx}\,dx & \left[\, x=g^{-1}(u) \,\right] \end{array}$$

Substituição trigonométrica:

$$\begin{array}{lll} \int_{s=a}^{s=b} F(s,\sqrt{1-s^2})\,ds & \int F(s,\sqrt{1-s^2})\,ds & \\ = \int_{\theta=\operatorname{arcsen} a}^{\theta=\operatorname{arcsen} b} F(\operatorname{sen}\theta,\sqrt{1-\operatorname{sen}^2\theta})\frac{d\operatorname{sen}\theta}{d\theta}\,d\theta & = \int F(s,\sqrt{1-s^2})\frac{ds}{d\theta}\,d\theta & \left[\begin{array}{c} s=\operatorname{sen}\theta \\ \theta=\operatorname{arcsen}\theta \end{array}\right] \\ = \int_{\theta=\operatorname{arcsen} a}^{\theta=\operatorname{arcsen} b} F(\operatorname{sen}\theta,\cos\theta)\cos\theta\,d\theta & = \int F(s,c)c\,d\theta & \left[\begin{array}{c} s=\operatorname{sen}\theta \\ c=\cos\theta \\ \theta=\operatorname{arcsen}\theta \end{array}\right] \end{array}$$

$$\begin{array}{lll} \int_{z=a}^{z=b} F(z,\sqrt{z^2-1})\,dz & \int F(z,\sqrt{z^2-1})\,dz & \\ = \int_{\theta=\operatorname{arcsec} a}^{\theta=\operatorname{arcsec} b} F(\sec\theta,\sqrt{\sec^2\theta-1})\frac{d\sec\theta}{d\theta}\,d\theta & = \int F(z,\sqrt{z^2-1})\frac{dz}{d\theta}\,d\theta & \left[\begin{array}{c} z=\sec\theta \\ \theta=\operatorname{arcsec} z \end{array}\right] \\ = \int_{\theta=\operatorname{arcsec} a}^{\theta=\operatorname{arcsec} b} F(\sec\theta,\tan\theta)\sec\theta\tan\theta\,d\theta & = \int F(z,t)zt\,d\theta & \left[\begin{array}{c} z=\sec\theta \\ \theta=\operatorname{arcsec} z \\ t=\tan\theta \end{array}\right] \end{array}$$

$$\begin{array}{lll} \int_{t=a}^{t=b} F(t,\sqrt{1+t^2})\,dt & \int F(t,\sqrt{1+t^2})\,dt & \\ = \int_{\theta=\arctan a}^{\theta=\arctan b} F(\tan\theta,\sqrt{1+\tan^2\theta})\frac{d\tan\theta}{d\theta}\,d\theta & = \int F(t,\sqrt{1+t^2})\frac{dt}{d\theta}\,d\theta & \left[\begin{array}{c} t=\tan\theta \\ \theta=\arctan t \end{array}\right] \\ = \int_{\theta=\arctan a}^{\theta=\arctan b} F(\tan\theta,\sec\theta)\sec^2\theta\,d\theta & = \int F(t,z)z^2\,d\theta & \left[\begin{array}{c} t=\tan\theta \\ \theta=\arctan t \\ z=\sec\theta \end{array}\right] \end{array}$$