Sheaves for Children - handouts Seminário Carioca de Lógica, 2014may19 Eduardo Ochs

ZHAs - examples

0	0		y=10
0 0	0	o o < ¬ZHA2	y=9
0 0 0	0	< ¬ZHA1	y=8
0 0 0	0 0	o < ¬ZHA3R	y=7
0 0 0	0	o o < ¬ZHA4	y=6
0 0 0	0 0	0 0	y=5
0000	0 0	0	y=4
0 0 0 0	0	0 \	y=3
0 0 0	0	o o < ¬ZHA5	y=2
0 0	0 0	00	y=1
0	0	o /	y=0

A ZHA (D_1) A ZHA (D_2) Not a ZHA (D_3)

A ZSet D is a ZHA iff it obeys these conditions: ZHA1: The non-empty lines of D are sequential ZHA2: D has a top element and a bottom element ZHA3L: The left wall of D can be traversed by black pawns moves ZHA3R: The right wall of D can be traversed by black pawns moves ZHA4: Each line of D is made of consecutive same-parity points ZHA5: All points in each wide region of D have the same parity

D_3 violates all the ZHA conditions... -ZHA1: The non-empty lines of D_3 are not sequential - y=8 is a gap -ZHA2: D_3 has more than one top element -ZHA3R: this is not a black pawn's move (from y=7 to y=6) -ZHA4: This line is not made of consecutive same-parity points (y=6) -ZHA5: This wide region has points of opposite parities

В	В	y=10				
0 0	В	y=9				
0 0 0	В	y=8 `	\			
0 0 0	0 0	y=7		(6,8)	in	DCB_D_2
0 0 0	В	y=6 ∖ ,	/			
0 0 0	0 0	y=5				
0 0 0 0	0 0	y=4		(3,6)	in	DCB_D_2
0 0 0 0	В	y=3 /				
0 0 0	В	y=2 `	\			
0 0	0 0	y=1		(0,2)	in	DCB_D_2
В	В	y=0	/			
ZHA with one	A ZHA wi	th				

A ZHA with one A ZHA wide region three

three wide regions 1

В	В
LR	В
LoR	В
LoR	LR
LoR	В
LoR	LR
LooR	LR
LooR	В
LoR	В
LR	LR
В	В

The left wall and the right walls of D_1 and D_2 ; the bottlenecks belong to both walls

0	G	
0 0	G	
Gоо	0	G
Gоо	GG	G
ооG	0	\ . G
0 0 0	Gо	. \ /
оооG	GG	X G
GooG	G	G . / G
GoG	0	G / G
GG	GG	GG
0	0	

The generators of D_1 and D_2, and the inter-wall arrows of D_1 $\,$

The intuitionistic implication:

$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Р	Q	P&Q	P\/Q	P->Q
1 I 1 M 1	 0 0	0	0	0	1
$= (10) \\ 1$	1	1 0 1	0 0 1	1 1 1	1 0 1
$= \begin{array}{c} 0\\ 1\\ 0\\ 1 \end{array}$					